Charles A. Oliver.

AN

ELEMENTARY TREATISE

ON

OPTICS.

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ADVERTISEMENT.

In preparing the following treatise, the writer has consulted the principal French and English works on optics and general physics, and selected from them whatever he thought adapted to his purpose. The materials thus obtained, he has attempted to remould and combine in such a manner as to furnish a clear and systematic exposition of the general subject. The only merit the case admits of, is that which may be claimed for a judicious choice of matter and methods, and the reduction of them to a uniform and consistent whole. The works to which the writer is chiefly indebted, are those of Pouillet, Despretz, Lamé, Biot and Fischer, Airy, Lloyd, Herschel and Coddington.

The writer would do violence to his feelings, should he conclude this brief account of his labors without acknowledging his obligations to his friend, Mr. John Paterson, of Albany, who has read the work in manuscript, and from whom many valuable suggestions have been received.

In the revision for the present edition, the treatises of Billet, Daguin and Jamin have been consulted, and alterations and additions, rendered necessary by the advancement of the science, have been made.

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OPTICS.

INTRODUCTION.

1. Two theories have been proposed to account for the phenomena of light: the theory of emission, and the wave-theory. In the former, the phenomena are conceived to be due to the emission of exceedingly minute particles of matter from the luminous body; in the latter, it is assumed that all space is pervaded by a medium of extreme tenuity; that luminous bodies possess the quality of exciting vibrations in this medium, and that these vibrations constitute light.

In the following work, the more common phenomena of light will first be considered without reference to either theory, though the language of the former may occasionally be used for the sake of convenience.

2. Bodies which possess the property of exciting the sensation of light, are said to be *luminous*.

3. Certain bodies possess in themselves this property, and are hence called *self-luminous;* as the sun, stars, flame, red-hot iron, etc.

4. Bodies not luminous of themselves, are capable of receiving light from self-luminous bodies and emitting it as their own, and of thus becoming luminous.

5. Any space or matter through which light may be transmitted, is called a *medium.*

6. In a homogeneous medium, light is *transmitted* or *propagated* in straight lines. A sufficient proof of this is, that if at any point of the straight line drawn between the eye and the object from which the light *emanates* or *radiates*, an opaque body be interposed, the object will cease to be visible.

7. A luminous body transmits light in all directions; for it is visible in all positions of the eye, if the light be not intercepted.

8. A luminous body may evidently be considered a collection of luminous particles or points, each point transmitting light in all directions.

9. Any straight line, in the direction of which light is transmitted, is called a *luminous ray.*

10. A luminous point may be considered as the centre of a sphere of luminous rays.

11. A collection of contiguous rays, emanating from a luminous point, is called a *pencil of light.*

12. If the distance of the luminous point be so great, compared with the dimensions of the base of

the pencil, that the rays may be considered sensibly parallel, the collection is called a *beam of light.*

13. The rays of a pencil of light, though naturally divergent, may be so modified as to become convergent, that is, be made to pass through a common point. This point of meeting of all the rays of a pencil, is called the *focus* of the pencil. It must be remarked, however, that after meeting at the focus the rays pursue their rectilinear course, and again become divergent, like a natural pencil.

14. *The intensity of the light emanating from a luminous point, varies inversely as the square of the distance.*

This law, which is here given as the result of experiment, is an easy deduction from the theory of emission.

The intensity of the light received by an illuminated surface, may be defined to be the absolute quantity of light which falls upon the unit of surface. Let the luminous point be the centre of a spherical surface: denote this surface, or any part of it, by s; the light which falls upon s, by q, and the intensity by I: then

$$s : q :: 1 : I$$

and

$$I = \frac{q}{s}.$$

Now let the luminous point be the common centre of two spherical surfaces s and s', of which the radii are r and r'; then, denoting the intensities by I and I′

$$I = \frac{q}{s} \text{ and } I' = \frac{q}{s'};$$

and hence $$I : I' :: \frac{q}{s} : \frac{q}{s'},$$

or $$I : I' :: s' : s;$$

but $$s' : s :: r'^2 : r^2;$$

hence $$I : I' :: r'^2 : r^2.$$

This law is also a consequence of the wave-theory. It may not be unnecessary to remind the student, that the transmission is here supposed to take place in a vacuum.

15. *The intensity of the light received by a plane surface, placed obliquely to the direction of the rays, is proportional to the sine of the angle which their direction makes with the surface.*

Let the beam of light pp' [Fig. 1] fall upon a plane surface bc, at right angles to its direction: using the same notation as before, we have

$$I = \frac{q}{s}.$$

If now the surface be inclined so as to make with the direction of the rays any angle a, we have (denoting the surface in its new position by s', and the intensity by I′),

$$I' = \frac{q}{s'}.$$

But $bc = dc \,.\, \sin a$; hence $s = s' \,.\, \sin a$,

and $$s' = \frac{s}{\sin a};$$

substituting this value of s' in that of I', we have

$$\mathrm{I}' = \frac{q}{s} \sin a,$$

or

$$\mathrm{I}' = \mathrm{I} \,.\, \sin a.$$

16. The angle which a ray of light makes with the surface from which it emanates, is called the *angle of emanation.*

The intensity of light emanating from a luminous surface, is proportional to the sine of the angle of emanation.

A luminous surface is found by experiment to be equally bright at all angles of emanation; hence we may consider the beam pp' as containing the same quantity (q) of light, whether the surface from which it emanates be at right angles to the direction of the rays, as in the position bc, or make with them any angle a, as in the position dc: the extent of the luminous surface being supposed to increase so as to keep the *dimensions of the beam the same.*

It is evident, that in this case, the proper measure of the intensity is the absolute quantity of light emitted by the unit of surface. This being admitted, and using the same notation as in the preceding articles, it may be shown that the intensity of the light emitted by the surface in the position bc is given by the equation

$$\mathrm{I} = \frac{q}{s};$$

whilst in the position dc, it is given by the equation

$$\mathrm{I}' = \frac{q}{s} \sin a,$$

or $$I' = I \, . \sin a.$$

Hence the truth of the proposition.

Instruments for measuring the intensity of light are called *photometers*.

17. Light moves in space with a uniform velocity of about 192,000 miles in a second of time. This fact is established by observations upon the eclipses of Jupiter's satellites.

18. If an opaque body be illuminated by a single luminous point, and the point be considered the vertex of a conic surface tangent to the body, that part of the surface which lies beyond the body will enclose a space into which the light cannot penetrate. This space is called the *shadow* or *umbra*. If there be a number of luminous points forming a luminous body of sensible dimensions, besides the umbra, there will be a portion of space adjacent to it partially deprived of light, called the *penumbra*. [See Fig. 2.]

19. When the sun's rays are admitted into a dark chamber by a very small aperture, and are intercepted at a suitable distance by a screen *at right angles to their direction*, there appears on the screen a *circular image of the sun*.

If the light emanated from a single luminous point, the form of the image would evidently depend upon that of the aperture. Thus, if the aperture were triangular, a pencil of light emanating from any point of the sun's disk would give a triangular

image. In this case the solar image would be made up of an infinite number of superimposed triangular images. The images formed by the beams emanating from the sun's circumference would be arranged circularly, and hence the contour of the whole would be circular.

In the same manner may be explained the formation of the images of the sun, which are sometimes observed under the dense shade of trees, circular or elliptical, according to the inclination of the rays; and also the formation of the inverted images of external objects, seen on the wall of a dark chamber when the light is admitted by a small aperture.

The reason of the inversion of the images is apparent from figure 3.

As a beam or pencil of light enters the aperture from every point of the sun or other object, it is evident, that what we have called the image, is the intersection, by the screen, of an infinite number of beams or pencils, more or less inclined to each other. Such a collection of beams or pencils may be called a *compound beam* or *pencil.* The term image is here used in a popular sense : *a true optical image is a collection of foci.*

Figs. 3 and 4 will illustrate the preceding remarks.

20. When a pencil of light LI [Fig. 5], moving in a homogeneous medium, arrives at the surface AB of a new medium, it undergoes important changes. It is separated into parts.

One of these parts IL′ is *regularly reflected*, or thrown back into the medium which it was just leaving.

A second part IR enters the new medium, and undergoes a change of direction according to a certain law, or is *regularly refracted.*

A third part is scattered in all directions about the point I: one portion of it *mnh* undergoing an *irregular reflection*, while the remaining portion *m′n′h′* enters the new medium.

Of the total amount of light which enters the new medium, a part, more or less considerable, is progressively *absorbed* or lost, as the rays penetrate into its substance. In opaque bodies the absorption is total, and takes place within a space less than we can appreciate. But even here it is highly probable that it is not effected abruptly, but requires for its completion a finite, though exceedingly short period of time.

Light, in undergoing either reflection or refraction, acquires, in a greater or less degree, peculiar properties. The act by which these properties are acquired, is called *polarization*, and the light possessing them is called *polarized light.*

Light which does not possess these properties, is called *unpolarized light.*

PART FIRST.

OF UNPOLARIZED LIGHT.

CHAPTER I.

OF CATOPTRICS, OR THE REFLECTION OF LIGHT.

21. If a small beam of solar light, admitted into a dark chamber, be incident upon a plane metallic mirror, highly polished: then,

§ 1. One portion of the incident beam will be reflected in a determinate direction; and the eye, placed anywhere in this direction, will perceive a brilliant image of the sun.

The rays forming this image are said to be *regularly reflected.*

§ 2. Another portion of the incident beam will be reflected in every direction, rendering the part of the mirror on which it falls visible from all parts of the room. The rays thus scattered are said to be *irregularly reflected.*

For the present we shall confine our attention to the regularly reflected rays.

Let AB [Fig. 5] be the surface of the mirror, LI any ray of the incident pencil, and IL′ the corre-

sponding regularly reflected ray; and at I the point of incidence, let NN′ be drawn perpendicular to AB.

The angle NIL which the incident ray LI makes with the perpendicular, is called the *angle of incidence.*

The angle NIL′ which the reflected ray IL′ makes with the perpendicular, is called the *angle of reflection.*

The plane of the angle of incidence, is called the *plane of incidence*; and the plane of the angle of reflection, the *plane of reflection.* Each of these planes is obviously perpendicular to the reflecting surface.

Adopting these definitions, the laws of the reflection of light from a plane surface may be thus enunciated:

§ 1. *The planes of incidence and reflection coincide.*

§ 2. *The angles of incidence and reflection are equal.*

The truth of the preceding laws may be proved by directly measuring the angles of incidence and reflection, and ascertaining the position of their planes; but astronomical observation furnishes the following far more accurate method.

Let TO [Fig. 6] be a telescope, movable about a horizontal axis at C: the axis TO of the telescope will describe a vertical circle, having its centre at C. In determining the altitude of a star, astronomers first observe it by direct light SO; and then,

changing the position of the telescope, by a beam of light S′IL reflected from the surface AB of mercury.

The angles SCN′, LCN′ which the axis of the telescope in its two positions makes with the vertical radius N′C, are invariably found to be equal. But the verticals NI, N′C are parallel; so also are the rays SC, S′I which come from the same star; hence the angles S′IN, RIN are equal to SCN′, LCN′ respectively, and consequently equal to each other. It is also evident that the plane of incidence S′IN coincides with the plane of reflection RIN.

To the above laws there are no exceptions. They are true for artificial as well as natural light, and are independent of the substance of the reflecting surface.

22. As to the *intensity* of the light which is regularly reflected, it may be stated, in general terms, as the results of experiment :

§ 1. *That it increases with the polish of the mirror.*

§ 2. *That it is least when the angle of incidence is nothing, and increases with that angle.**

§ 3. *That it varies with the nature of the reflecting substance, and also with the medium in which the reflection takes place.*

* A change in the angle of incidence does not produce the same effect in all cases. Thus while in reflection by water or glass, the intensity increases rapidly with the angle of incidence, in reflection by mercury it increases very slowly.

At an incidence of 15°, water reflects about 2 per cent of the incident light, mercury about 60 per cent.

23. We now proceed to the application of the laws of reflection to the formation of images by mirrors.

Plane Mirrors. Let MM′ [Fig. 7] be a plane mirror, L a luminous point, and LI any ray emanating from it; and let IL′ be the corresponding reflected ray. Draw LOF perpendicular to MM′, and produce L′I meeting it in the point F. Draw also NN′ perpendicular to MM′. Then since the angles LIN, L′IN of incidence and reflection are equal, we have OF = OL. Whence it appears that all the rays emanating from L, and falling upon the mirror, will be reflected in lines which, produced backwards, will pass through the same point F. Since then an object is always seen in the direction in which the rays proceeding from it enter the eye, to an eye placed anywhere within the limits of the reflected pencil, the rays will seem to diverge from F, and the luminous point will appear to be at F on the perpendicular LF, and as far behind the mirror as it actually is in front of it.

The point F is called the *virtual focus* of the pencil LII′; the term *virtual* being employed to distinguish it from a focus, at which there is a *real concentration of light.* It will be perceived that the points L and F are symmetrically situated with respect to the mirror.

Now, if instead of a single luminous point L [Fig. 8], we have a finite object, as a straight line

LL$'$ placed before the mirror, the light proceeding from each point, as p will have a focus p', determined by drawing pp' perpendicular to the mirror, and making $ap' = ap$. There will thus be a series of foci, arranged in the present case in a straight line. This assemblage of foci is called the *image of the line* LL$'$. The object and image are evidently *equal, and symmetrically situated with respect to the mirror.*

It is found that the more perfect the polish of a mirror, the more vivid the image, and the less the intensity of the light irregularly reflected. Hence, were the polish of a mirror absolutely perfect, there would be no irregular reflection : the mirror would be invisible, and we should see only the image of the luminous object. The regular reflection, by which optical images are formed, is called *specular reflection ;* the irregular reflection, by which bodies are rendered visible, *radiant reflection.*

24. *Goniometers.* The laws of the reflection of light have been applied to measuring the *diedral angles* of polished bodies, and particularly of *crystals.* The instruments used are called *goniometers.* We will briefly explain the principle upon which they are constructed.

Let it be required to measure the diedral angle at A [Fig. 9] of the crystal ABC.

First, let the position of the crystal be such, that the edge of the diedral angle at A may be parallel to two distant, parallel, and horizontal lines D and

D′ (the edge and the lines being supposed to be perpendicular to the plane of the paper); and that the ray of light DI proceeding from the line D may, after reflection, coincide with a direct ray D′IO from the line D′. Now let the crystal revolve about the edge at A, until the surface AC shall coincide with MN, the plane of the first surface AB. Then it is evident that the ray from D will, after reflection from the surface AC in its new position, coincide as before with the direct ray from D′; and that during the revolution, the surface AC must have revolved through the angle CAC′.

But the angle required, viz. BAC = 180° — CAC′; hence the problem is reduced to determining the angle through which the crystal has revolved.

The instrument invented by Dr. Wollaston for this purpose consists of a graduated vertical circle, capable of motion about a horizontal axis AB [Fig. 10], through which passes a smaller axis *cd*. The extremity of this smaller axis is furnished with several joints, by means of which the position of the plate *e*, on which the crystal to be examined is fixed, may be changed at pleasure. To use this goniometer, a building is selected, at the distance of sixty or eighty yards from the observer, which presents several parallel and horizontal lines; and such a position is given to the instrument, and also to the crystal, that the edge of the angle required,

the axis, and the lines to be used as sights, may all be parallel.

Then, starting from a position in which one of these lines seen directly, coincides with another seen by reflection from one of the faces of the crystal, the greater axis is turned, and with it the crystal, till a like reflection and coincidence take place on the adjacent face. The angle through which the crystal has revolved may be read off on the graduated circle.

It will be perceived that the great distance of the sights, compared with the dimensions of the crystal, is essential to the exactness of the admeasurement.

25. *Parallel Plane Mirrors.* Since in the formation of optical images by plane mirrors, the pencils of light are reflected from the mirror, just as if they diverged from the different points of the image, we should expect that, if they fell upon a second mirror, a second image would be formed, having the first image as its object; that this second image would produce a third; the third, a fourth, and so on.

This conclusion is verified by experiment. Thus, if an object [Fig. 11] be placed between two parallel mirrors M and M′, a series of images, diminishing in distinctness as they recede from the mirrors, will be formed by each, and may be seen by an eye placed in a suitable position.

Here, the rays which fall directly upon the two mirrors form their respective images at A and A′. The rays which form the image at A are reflected from M as if they came from A, and, falling upon the mirror M′, form an image at C; the two images being symmetrically placed with reference to the mirror M′. Again, the rays reflected from M′, and appearing to emanate from C, fall upon the mirror M, and form an image at D, the two images at C and D being symmetrically placed with respect to the mirror M; and so on indefinitely.

Since the intensity of the reflected pencils is diminished by the successive reflections, the images ought to appear less vivid, as the distances at which they are formed become greater.

26. *Inclined Plane Mirrors.* When the two mirrors are inclined to each other, there is a similar multiplication of images, the number of them depending upon the angle of the mirrors.

We will examine the case in which the mirrors make a right angle.

Let MC and M′C [Fig. 12] be the mirrors, and A the object. With any radius CA, describe a circle ABDB′; draw AB and AB′ perpendicular to CM and CM′: B and B′ will evidently be the positions of the images formed by the light which falls directly upon the mirrors from A. Again: drawing BD, B′D perpendicular to the directions of the mirrors produced, the point D will be the position of the

images symmetrical with B and B′, the two images coinciding.

If the object were a straight line GH, the images would be EH, FE, and FG; which, together with the object, would form a symmetrical figure EFGH.

If the angle of the mirrors were 60°, there would be five images; if 45°, seven; or, if we consider the object an image, in the former case six, in the latter eight; and in general, if the object be reckoned with the images, in order that there may be 5, 6,........20, etc. images of the same object, the angle of the mirror must be equal to $\frac{1}{5}$, $\frac{1}{6}$,..........$\frac{1}{20}$, etc. of the whole circumference.

27. *Spherical Mirrors.*

§ 1. In the reflection of light by curved surfaces, it is found, that *the reflection of each ray is effected as if it took place upon the plane tangent to the surface at the point of incidence.* The preceding laws are, therefore, applicable to the present case, if we regard the light as incident upon the tangent plane, instead of the surface itself.

§ 2. The curved surfaces usually employed, are *spherical.*

§ 3. In order to the formation of distinct images by spherical mirrors, it is found necessary in practice that the rays should fall upon the reflecting surface *almost at right angles.*

§ 4. Let BB′ [Fig. 13] be an arc of a circle, and from its centre C draw CA bisecting it. If this arc

be revolved about AC as an axis, it will describe a segment of a spherical surface, which may be considered the surface of a spherical mirror, *concave* or *convex*, according as the light is supposed to fall upon the inner or outer surface.

The radius AC produced indefinitely, is called the *axis of the mirror;* C, its *centre of curvature;* and A, its *centre of figure.*

The angle BCB′ is called the aperture of the mirror. From the restriction requiring the rays to be almost perpendicular to the surface of the mirror, it follows that this angle must be always very small. It is found that it should not exceed six or eight degrees.

28. We proceed now to consider the reflection of a luminous pencil, emanating from a point, placed anywhere in front of the mirror.

The point may be situated on the axis of the mirror, or without it. We shall first consider the case in which *it is on the axis.*

§ 1. Let the point be at an infinite distance from the mirror; the rays that fall upon it, will then be sensibly parallel to the axis.

Let BB′ [Fig. 14] be a section of a concave mirror by a plane passing through the axis; and let OM, parallel to AC, be one of the incident rays. Draw the radius MC: it will be perpendicular to the tangent at M; hence, if in the plane CMO we draw

MF, making the angle CMF = CMO, MF will be the reflected ray.

Now OMC = MCF : hence CMF = MCF, and MF = FC. But since AM is a very small arc, MF is very nearly equal to AF, and hence AF very nearly equal to FC, or F is very nearly the middle point between A and C.

The same may be shown to be true of any parallel ray in this, or any other section of the mirror. Hence we may say, without sensible error, that the rays which fall upon the mirror, parallel to the axis, are so reflected as to converge to a point half-way between the two centres A and C.

This point is called the *principal focus*, and AF the *principal focal distance*. At this focus there is a real concentration of light. In like manner it may be shown, that rays falling upon a convex mirror [Fig. 15] parallel to the axis, will be reflected so as to *appear* to diverge from a point F behind the mirror, half-way between A and C. At this point, there is not an *actual* concentration of the rays of light; hence it is called the *virtual principal focus*, and AF the *virtual principal focal distance.*

§ 2. Let the luminous point be at a finite distance from the mirror.

Then MM′ [Fig. 16] being, as before, the section of a concave mirror, AP the axis, and P the position of the point, PM the incident ray, P′M the reflected ray, and MC the radius, we have

$$CMP = MCA - MPC,$$

and
$$CMP' = MP'A - MCA;$$

and hence
$$MCA - MPC = MP'A - MCA,$$

or
$$2\,MCA = MPC + MP'A \quad \ldots\ldots\ldots\ldots [A.]$$

Now as AM is a very small arc, we may consider it a straight line, perpendicular to AP; hence, we have

$$\frac{AM}{AP} = \tan MPC, \quad \frac{AM}{AC} = \tan MCA, \text{ and } \frac{AM}{AP'} = \tan MP'A.$$

But as the acute angles at P, C, and P′, are necessarily very small, we may substitute for their tangents the angles themselves; hence we have

$$\frac{AM}{AP} = MPC, \quad \frac{AM}{AC} = MCA, \quad \frac{AM}{AP'} = MP'A;$$

and hence, finally, by substituting these values in equation [A], we get

$$\frac{2}{AC} = \frac{1}{AP} + \frac{1}{AP'};$$

or denoting the principal focal distance by f, and the distances AP and AP′ by p and p' respectively,

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{p'} \quad \ldots\ldots\ldots\ldots\ldots\ldots [B.]$$

Since AM is eliminated from this equation, and p', or the position of the point P′, is determined solely by p (f being constant for the same mirror), it is evident that all rays emanating from P, and making small angles with the axis, sensibly converge after reflection to the same point P′ on the axis, and there form an image of the point P. This

image will be perceived by an eye so placed as to receive the rays which, after intersecting at P′, form a pencil diverging from it.

P′ is, then, the focus of the rays which emanate from P; but if we suppose the luminous point to be at P′, the rays will, after reflection, converge to P. These points are hence called *conjugate foci.*

29. We will now suppose the luminous point P″ [Fig. 17] to be *without the axis.*

This case is immediately reducible to the preceding. For, through the point and centre of curvature draw the straight line P″A′; it is clear that this line may be considered as an axis to the mirror, and that the relation between the conjugate foci P″ and P‴ is expressed by the equation

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{p'},$$

p and p' denoting the distances A′P″, A′P‴.

Hence, all rays emanating from P″, and falling upon the mirror, will, after reflection, meet the axis A′C sensibly in the same point P‴.

It must be remarked, however, that the condition of distinct images [Art. 27, § 3] requires the point P″ to be relatively near the axis. The axis AC is called the *primary axis;* any other, as A′C, a *secondary axis.*

30. The equation $\frac{1}{f}=\frac{1}{p}+\frac{1}{p'}$, though originally investigated for the case of a concave mirror, is entirely general. Thus, considering the quantities

p, p' and f as positive when reckoned from A to the right, and negative when reckoned from the same point in the opposite direction, to adapt it to a convex mirror, we have only to make f negative; to converging rays, to make p negative; and according as p' is affected with the positive or negative sign, the focus lies to the *right* of the mirror, and is *real*, or to the *left*, and is *virtual*.

We will examine the most important cases.

§ 1. The *mirror concave*, and the *rays diverging*.

For this case we have

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{p'};$$

$$\frac{1}{p'} = \frac{1}{f} - \frac{1}{p} = \frac{p-f}{pf},$$

and
$$p' = \frac{pf}{p-f} = \frac{f}{1-\frac{f}{p}}.$$

‡ 1. Let $p < f$; then p' is negative, and the focus is virtual.

When $p = 0$, we have $p' = 0$; that is, when the point touches the mirror, it and its image coincide.

As p increases, $\frac{f}{p}$ diminishes; $\left(1 - \frac{f}{p}\right)$ also diminishes, and p' increases, remaining constantly negative. That is, while the point moves from A towards the principal focus, its image moves from the same point in the opposite direction.

‡ 2. Let $p = f$; then $p' = \infty$.

That is, when the point arrives at F, its image is at an infinite distance from the mirror, or rays

which emanate from the principal focus are reflected parallel to the axis. [Fig. 18.]

‡ 3. Let $p > f$; then p' is always positive, and the focus is real.

Now, as p increases, $\frac{f}{p} < 1$ diminishes, $\left(1 - \frac{f}{p}\right)$ increases, and p' constantly diminishes; and when $p = 2f$, $p' = 2f$; and when $p = \infty$, $p' = f$. That is, as the point moves from the principal focus to the centre, the image moves from an infinite distance, in front of the mirror, to the same point, and the two coincide. The point continuing to recede from the mirror, the image passes the centre, and is formed at the principal focus, when the distance of the point becomes infinite.

If we make $p = 1000f$, we shall have $p' = f + \frac{f}{999}$; and if f be equal to one foot, the image will be formed only $\frac{1}{999}$ of a foot from the principal focus. When, therefore, the point attains a considerable distance from the mirror, as one thousand times the principal focal distance, for instance, the image is formed sensibly at the principal focus.

§ 2. The *mirror convex*, and the *rays diverging*. Here f is negative, and we have

$$p' = -\frac{pf}{p+f} = -\frac{f}{1+\frac{f}{p}}, \text{ constantly } \textit{negative}.$$

When $p = 0$, we also have $p' = 0$: as p increases, $\frac{f}{p}$ diminishes, and p' increases; and when $p = \infty$, $p' = -f$.

That is, whilst the point moves from the mirror to an infinite distance in front of it, the image moves only from the mirror to the virtual principal focus behind it.

It is evident that the preceding discussion is as applicable to a secondary axis as it is to the principal axis.

31. If, now, we suppose the luminous object to be a body of finite dimensions, there will be a collection of foci, constituting its image, the position and magnitude of which may be determined by the foregoing principles. The subject may be simply illustrated by the following geometrical constructions :

Let MM′ [Fig. 19] be a concave mirror, AC its axis, F the principal focus, and C the centre of curvature. Let the object be a straight line perpendicular to the axis, and bisected by it.

§ 1. Let the object PP′ be at the principal focus. The ray PM emanating from the top of the object, parallel to the axis, will be reflected so as to pass through F ; and a ray PA falling upon the mirror at its centre, will be reflected so as to pass through P′, the lowest point of the object. Considering AM as a straight line perpendicular to the axis, it may easily be shown that MF and AP′ are parallel, and hence that the image is at an infinite distance.

§ 2. Let the object P″P‴ [Figs. 19 and 20] be between the mirror and the principal focus. Then,

comparing figures 19 and 20, since $P''AF$ is greater than PAF, we have $P'''AF$ greater than $P'AF$; and hence the rays MF and $P'''A$ produced, will meet at some point I behind the mirror. Thus, the pencil of rays emanating from P'' will be so reflected as to appear to issue from the point I. This point, then, is the image of the point P''. II' [Fig. 20] drawn perpendicular to the axis and bisected by it, is the image of $P''P'''$. It is upright, and larger than the object.

§ 3. Let the object $P^{iv}P^{v}$ [Figs. 19 and 21] be a little more distant from the mirror than F; then a comparison of figures 19 and 21 renders it evident that the rays will meet at some point I in front of the mirror; and the less distant from it, the greater the distance of the object from the principal focus; so that, as the object moves from F, the image moves towards it. The image II' [Fig. 21] is inverted, and larger than the object.

§ 4. Let the object $P^{vi}P^{vii}$ [Fig. 22] be at the centre of curvature. Then, since $AF = FC$, the ray MF will pass through the point P^{vii}; and the image will occupy the same place as the object, and be of the same size, but inverted.

§ 5. Let the object $P^{viii}P^{ix}$ [Figs. 22 and 23] be at a greater distance than AC; then, comparing figures 22 and 23, it is clear that the image will be between C and F, smaller than the object, and inverted.

A similar mode of construction is applicable to convex mirrors. [See Fig. 24.]

32. The preceding results may be *verified* by placing an object of the proper dimensions before a spherical mirror, and then varying its distance from the mirror.

If the mirror be *concave*, and the object in contact with it, the image is erect, and of the same size as the object, and formed on the same surface. As we move the object from the mirror, the image, remaining erect, increases in size, and appears to move in the opposite direction; when the object reaches the principal focus, the image disappears altogether. The motion of the object continuing, the image reappears in front of the mirror, distant, magnified, and inverted; and may be received on a screen, or may be seen without a screen by an eye placed in a suitable position. As the object continues to recede, the image, constantly diminishing, approaches the mirror, till, at twice the focal distance, that is, at the centre of curvature, the two appear to occupy the same place, and to be of the same size, the image being still inverted. As the object moves from the centre, the inverted image, becoming less and less, continues to move towards the mirror; but, however distant the object, it never seems to pass the principal focus. It, indeed, appears to attain that point when the distance of the object is very con-

siderable, compared with the dimensions of the mirror.

If the mirror be *convex*, the image is, in all cases, formed behind it, erect, and smaller than the object. While the object moves from the mirror, the image constantly diminishing, appears to move in the opposite direction; but never beyond the principal focus.

33. It is sometimes necessary to determine the principl focal distance of a mirror. The following method is sufficiently accurate for ordinary purposes.

When the mirror is *concave*, such a position is given to it, that its axis may be parallel to the solar rays, and the image of the sun is received upon a small screen of ground glass. The position of the screen, in which the image is least and best defined, determines the place of the principal focus; and the distance from it to the mirror, the quantity sought, may be found by careful admeasurement.

When the mirror is *convex*, its surface must be covered with some badly reflecting substance, in which there are two very small circular holes A and B [Fig. 25] symmetrically placed with respect to the axis. The mirror being disposed as before, with its axis parallel to the solar rays, a screen, pierced so as to admit the rays LA, L′B, must be moved backward and forward in front of it, till that position be found, in which the distance A′B′, between the points where the reflected rays intersect the

screen, is equal to twice AB. The distance from the screen to the mirror, is, then, equal to the principal focal distance.

34. In all that precedes, we have spoken of the focus F [Fig. 26] as a single point; but it is evident that this is not rigorously true, even for those rays which, like LM, LM′, fall very near the centre of the mirror. Still, in this case, the language is admissible; since the focus, if not a single point, is, relatively, a very small space. But, if we consider those rays which fall near the edge of the mirror, as LM″, LM‴, we shall have an entirely new focus at some point F′, nearer the mirror; and intermediate rays will give us intermediate foci.

This property of mirrors, by which the incident rays are reflected to different foci, is called the *spherical aberration of mirrors.* The distance FF′ is called the *longitudinal spherical aberration;* and depends upon the aperture of the mirror, the radius of curvature, and the distance of the luminous point.

In regard to the formation of images, it is evident that the effect of spherical aberration is to produce an ill-defined image; the focus of the mirror being, in fact, an assemblage of foci, and the image consisting of a series of images distributed along the line FF′. The remedy, obviously, is to diminish the aperture of the mirror.

35. It is easy to conceive that the form of a surface might be such as to cause the rays of light to converge accurately to one point; and such, indeed, is the form of the surfaces generated by the revolution of the conic sections about their axes.

Thus, if rays parallel to the axis of a parabolic mirror fall upon the concave surface of the mirror, they will be reflected into the focus of the generating parabola; if they fall upon the convex surface, they will be reflected so as to diverge from the same point.

Rays emanating from one of the foci of an ellipsoid, are reflected by the concave ellipsoidal surface into the other focus; rays converging to one of the foci, and incident upon the convex surface, diverge after reflection from the other focus.

The hyperboloid possesses the same property.

The cause of this accurate convergence is obvious, when we consider that the radii vectores, drawn to any point in a conic section, make equal angles with the tangent at that point.

This property of reflecting rays accurately to the same point, is possessed by no other surfaces than those above mentioned.

36. When the aperture of the mirror is large, and the form differs from that of any of the surfaces just considered, the rays are, as we have seen, reflected to different foci. Hence, the consecutive rays intersect each other. An assemblage of points is

thus determined, forming a curved surface, called *a catacaustic*, or *a caustic by reflection.*

If the reflection take place upon a line, instead of a surface, the caustic will be a line. Thus, parallel rays falling upon the circumference MN [Fig. 27] will, after reflection, intersect the axis in the points *a*, *b*, *c*, *d*, etc., and each other in the points *m*, *n*, *o*, *p*, etc., forming the caustic.

To this curve, the reflected rays are *tangents.*

CHAPTER II.

OF DIOPTRICS, OR THE REFRACTION OF LIGHT.

37. WHEN a ray of light passes from one medium into another, it, in general, undergoes a change of direction. This phenomenon is called *refraction.*

Let AB [Fig. 28] represent the common surface separating two media, and let a beam of light be incident upon it in the direction LI; then, that part of the beam which enters the second medium will not, in general, move in the direction of LI produced, but in some other direction, as IR.

Let NN′ be drawn perpendicular to AB at the point of incidence; then, NIL being the angle of incidence, the angle RIN′ which the refracted ray IR makes with the perpendicular is called the *angle of refraction.*

The plane of the angle of incidence is called the *plane of incidence;* and the plane of the angle of refraction, the *plane of refraction.*

The refracted ray is not always single. In certain cases, it is divided into two distinct rays. The latter phenomenon is called *double refraction*, to distinguish it from ordinary or simple refraction. We shall here consider simple refraction only.

38. The fundamental laws of simple refraction are two.

§ 1. *The planes of incidence and refraction coincide.*

§ 2. *The ratio of the sines of the angles of incidence, and refraction is constant for the same media.*

Let NP′N′ [Fig. 29] represent a vertical section of a hemispherical vessel of glass filled with water, so that NN′ the surface of the liquid may pass through the centre I; and PNP′N′ a vertical graduated circle, in the same plane, attached to the vessel.

Now, if, in the plane of this circle, we direct a ray LI of solar light to the centre I, it will be found, 1st, that the refracted ray IR remains in the plane of the circle; and 2d, denoting the angles of incidence and refraction (measured by the arcs PL, P′R of the graduated limb) by I and R,

that $$\frac{\sin I}{\sin R} = \frac{LD}{RF} = \frac{4}{3}.$$

Again, if we vary the angle of incidence, so that the incident and refracted rays become L′I, IR′, we shall still find that the planes of incidence and refraction coincide, and that

$$\frac{\sin I}{\sin R} = \frac{L'D'}{R'F'} = \frac{4}{3}.$$

And in general, at whatever angle the light may be incident, the planes of incidence and refraction will be found to coincide, and the ratio of the sines to be equal to the constant number $\frac{4}{3}$.

39. If, in the preceding experiment, some other liquid be substituted for the water, or another gas for the atmospheric air which surrounds the apparatus, or the density of either the air or the water be varied, it will be found, that while the ratio remains constant for the same media, it, in general, varies when the media are changed, or suffer any variation in their physical state.

This constant ratio of the sines is called the *index of refraction*. If we denote it by n, we shall have

$$\frac{\sin \mathrm{I}}{\sin \mathrm{R}} = n. \qquad \text{[C.]}$$

40. In the foregoing articles, the surface separating the media has been supposed to be a plane; but the above laws are found to be true when the media are separated by any curved surface whatever.

In this case, the light is considered as incident upon a plane tangent to the surface at the point of incidence.

41. If, in the equation

$$\frac{\sin \mathrm{I}}{\sin \mathrm{R}} = n,$$

we suppose n greater than unity, $\sin \mathrm{I}$ will be greater than $\sin \mathrm{R}$, and I greater than R; that is, the refracted ray IR will be inclined towards the perpendicular. In this case, the second medium is said to be *more refractive* than the first.

If $n = 1$, then $\mathrm{I} = \mathrm{R}$, and there is no refraction; and the two media are said to be *equally refractive*.

If n be less than unity, I is less than R, and the refracted ray is bent from the perpendicular; and the second medium is said to be *less refractive* than the first.

These results are usually enunciated by saying, that, as the second medium is more or less dense than the first, the light is bent towards or from the perpendicular. But this is not strictly true, as, in general, the refraction is not proportional to the density; and there are cases in which a medium, though less dense than another, is more refractive.

42. If the light, instead of passing from the air into the water, be made to pass from the water into the air, and at the same time be incident in the direction RI, it will be found to be refracted in the direction IL; that is, the light will pass from R to L, in the same path by which it came from L to R.

Now, n being the index of refraction, when the light passes from the air into the water, or from the first medium into the second, we have

$$\frac{\sin \mathrm{LIP}}{\sin \mathrm{RIP}'} = n,$$

or

$$\frac{\sin \mathrm{RIP}'}{\sin \mathrm{LIP}} = \frac{1}{n}.$$

But, in this case, RIP′ is the angle of incidence, and LIP the angle of refraction; hence, denoting these angles as before by I and R, we have

$$\frac{\sin \mathrm{I}}{\sin \mathrm{R}} = \frac{1}{n}.$$

That is, the index of refraction for light passing from the first medium into the second being denoted by n, the index for the case in which it passes from the second into the first will be $\frac{1}{n}$, or the reciprocal of n.

43. Resuming the consideration of the equation

$$\frac{\sin I}{\sin R} = n,$$

if we suppose $I = 0$, we shall have $R = 0$; that is, when a ray of light falls perpendicularly upon the surface of the second medium, it suffers no refraction. This is confirmed by experiment.

Again, let $I = 90°$; then we shall have

$$\sin R = \frac{\sin I}{n} = \frac{1}{n}.$$

$I = 90°$ being the limit of the angle of incidence, the corresponding value of R must be the limit of the angle of refraction, and may be calculated when we know n. For the sake of brevity we shall call it the *limiting angle.*

In the case of light passing from air into water, in which $n = \frac{4}{3}$, we have

$$\sin R = \tfrac{3}{4};$$

and hence $R = 48° 35'$. That is, the limiting angle in this case is $48° 35'$.

Thus it appears that, while the ray PI [Fig. 30] increases in obliquity till it barely grazes P″I the surface of the water, the refracted ray also becomes more oblique, but much less rapidly than PI; and

never attains a greater obliquity, than in the position IR″, in which N′IR″ = 48° 35′.

From this, it appears that light, in passing from air into water, cannot penetrate the latter in a direction more oblique than IR″; so that, if AB [Fig. 31] represent the surface of a vessel ABCD filled with water, and light be prevented from entering the part AI by an opaque covering; then, not a single direct ray of light can penetrate the space AIR″, the angle N′IR″ being equal to 48° 35′.

Conversely, when the light passes from the water into the air, as the angle of incidence R′IN′ [Fig. 30] increases, the angle of refraction NIP′ also increases, but more rapidly than R′IN′; and when R′IN′ reaches the limit N′IR″, the transmitted ray emerges in a direction parallel to the surface of the water.

To learn what takes place when the angle of incidence becomes greater than the angle N′IR″, recourse must be had to experiment. From it we learn that the moment the limit N′IR″ is passed, the light is thrown back into the water, and *totally* reflected within it according to the ordinary law.

This is the only case in which light is reflected without any diminution of its intensity; and the phenomenon is hence called the *total reflection* of light.

From the total reflection of light, very singular consequences are deducible. Thus, if a cylinder of

glass terminated by two plane faces, the one perpendicular to the axis and the other inclined to it at an angle equal to the complement of the limiting angle, be turned directly towards the sun, so as to receive the solar rays upon the perpendicular face, the eye may be placed with impunity against the oblique face; for not a ray of solar light will be transmitted: all will suffer total reflection.

Again, if we suppose an observer immersed in water to have his eye at the point O [Fig. 32], all the rays which enter his eye from objects above the surface AB of the water will be comprised within the right cone IOI′, the angle IOI′ being equal to double the limiting angle. That is, he will see external objects only through a circular aperture having II′ for its diameter; but all objects from the horizon to the zenith will be visible within this space. Beyond this aperture the surface of the water will act as a reflector, in which he will perceive the images of subaqueous objects.

44. *Refraction of light transmitted through a medium bounded by parallel plane faces.*

Let ABCD [Fig. 33] be the medium, terminated by two parallel plane faces AB and CD; and let the medium below CD be the same as that above AB.

Let PII′R represent the direction of the incident, refracted, and emergent rays; and let NN′, N″N‴

be perpendiculars to the surfaces at the points I and I′; then we have

$$\frac{\sin \mathrm{PIN}}{\sin \mathrm{N'II'}} = n, \text{ and } \frac{\sin \mathrm{II'N''}}{\sin \mathrm{N'''I'R}} = \frac{1}{n}.$$

Multiplying these two equations together, and observing that $\mathrm{N'II'} = \mathrm{II'N''}$, we have

$$\frac{\sin \mathrm{PIN}}{\sin \mathrm{N'''I'R}} = 1,$$

or

$$\sin \mathrm{PIN} = \sin \mathrm{N'''I'R}.$$

That is, *the incident and emergent rays* PI and I′R are *parallel.* Experiment confirms this.

45. *Refraction of light in passing through successive media bounded by planes, all of which are parallel to each other.*

Let ABDC, CDFE [Fig. 39] be any two media, terminated by parallel plane surfaces; and let there be, above AB and below EF, a medium different from either. Let the several media be denominated the 1st, 2d and 3d, as represented in the figure.

It is proved by experiment that a ray incident upon the surface AB of the second medium, in any direction LI, and refracted at the points I, I′ and I″, emerges from the surface EF of the third medium in a direction I″L′ parallel to LI. Let now another ray $\mathrm{QI}_{,}$ parallel to LI, be supposed to pass directly from the first medium into the third, and to emerge from the third in the direction $\mathrm{I}_{,,}\mathrm{Q}'$. By the preceding article $\mathrm{QI}_{,}$ and $\mathrm{I}_{,,}\mathrm{Q}'$ will be parallel to each other, and hence $\mathrm{I}_{,,}\mathrm{Q}'$ will be parallel to I″L′; but

the refractions at I'' and $I_{\prime\prime}$ are equal : consequently $I_{\prime}I_{\prime\prime}$ will be parallel to $I'I''$. It thus appears that in the transmission of light through three different media terminated by parallel planes, the course of the ray in the third medium, and hence the total deviation, is the same as if the intermediate medium did not exist, and the ray passed directly from the first into the third. It can hence be readily shown, that in the case of four successive media, the total deviation will be the same as if the light had passed directly from the first into the fourth ; and that in general, whatever the number of media, the deviation will be the same as if the light had passed directly from the first medium into the last.

46. *Refraction of light by transmission through prisms.*

§ 1. In optics, any medium having two plane surfaces inclined to each other at any angle, is called *a prism.*

§ 2. The *edge* of the prism is the line in which the two plane surfaces meet, or would meet if produced.

§ 3. The *base* of the prism is the surface opposite the edge, whether really existing, or only supposed to exist.

§ 4. The *refracting angle* of the prism is the angle made by the two plane surfaces.

§ 5. A *principal section,* is a section made by a plane perpendicular to the edge.

§ 6. The prisms generally used in optical experiments, have three rectangular surfaces. Their principal sections are triangles; and the prisms are said to be *rightangled*, *isosceles*, *equilateral*, or *scalene*, according to the form of these triangular sections.

Let ASA′ [Fig. 34] be a principal section of a glass prism, and LI a ray incident in the plane of the section. Since the planes of incidence and refraction coincide, the ray will pass through the prism, and emerge from the second surface in this same plane; and as the glass refracts more powerfully than the air by which it is surrounded, it will pass through the prism in some direction II′ inclined towards the perpendicular NN′, and will emerge in some direction I′E bent from the perpendicular N″N‴.*

The change of direction which light undergoes by transmission through a prism is accompanied by a very singular phenomenon. Each ray of an incident pencil which enters the prism is separated into a number of other rays, which emerge divergent, and exhibit all the colors of the rainbow. This important circumstance will be considered hereafter: at present we shall continue to represent the refracted ray by a single straight line. Whatever may be proved generally true of it, may be considered true of each one of the refracted rays into which the single incident ray is separated.

*See note to this article at the end of the chapter.

47. *Conditions of emergence.*

Since light, when passing from one medium into another of less refractive power may fall so obliquely upon the common surface as to suffer internal reflection instead of transmission, we shall consider for a moment the conditions of emergence.

Let L equal the limiting angle for the passage of light from the glass of the prism into air, and G the refracting angle of the prism. We shall examine only the cases in which

$$G = 2L, \quad G = L, \quad G < L.$$

§ 1. Let $G = 2L$. [Fig. 35.]

Here the ray incident at I and parallel to AI, will be refracted in the direction II′ making with the normal the angle $N'II' = L = \frac{1}{2}G = \frac{1}{2}ISI'$; so that II′ will be perpendicular to SM, which bisects the angle ISI′. Hence the ray will fall upon the second surface at the limiting angle. Every other incident ray, as LI, will fall upon the second surface in some direction II″ more oblique than II′, and hence will suffer total reflection. In this case, therefore, none of the light incident upon the surface AS can pass through the surface A′S. A chamber with a single aperture closed by such a prism, would be perfectly dark.

§ 2. Let $G = L$. [Fig. 36.]

Here it is evident that the incident ray, which coincides with the normal, will fall upon the second

surface at the limiting angle; and that all rays which fall in the angle AIN will emerge, while those which fall in the angle NIS will suffer total reflection.

§ 3. Let G < L. [Fig. 37.]

In this case, a little consideration renders it evident that all the rays incident in the angle AIN will emerge, and some also of those incident in the angle NIS.

48. *Of the angle of deviation, and its minimum value.* When the condition of emergence is fulfilled, the rays pass through the second surface, and undergo a greater or less change of direction, depending upon the angle of incidence, the index of refraction between air and glass, and the refracting angle of the prism.

ASA′ [Fig. 38] being, as before, a principal section of the prism, LI a ray incident in the section, II′ and I′O the corresponding transmitted and emergent rays, and O the place of the eye; let L′O be drawn so that, produced, it would pass through the point of the luminous object from which the ray LI proceeds; and let the distance of the object be so great, compared with the distance between the eye and the prism, that L′O may be considered parallel to LI; then, the change of direction which the light undergoes is evidently measured by the angle MO′O or its equal O′OL′. The angle MO′O is called the *deviation.* The point O is supposed to

be far enough from the prism to permit the observer to see the object by the direct light $L'O$ and the refracted light $O'O$ at the same time. This being the case, if, starting from a certain position, the prism be made to revolve constantly in the same direction about an axis parallel to its edge, the object, seen through the prism, will first appear to move in a certain direction, then to become stationary, and then to move again in the opposite direction. At the same time, the deviation will first diminish, attain its minimum when the object appears stationary, and afterwards increase. If, when the prism is in the position in which the deviation is a minimum, the angles of incidence and emergence be carefully measured, they will be found to be equal.

If we seek the position of minimum deviation by the calculus, we shall arrive at the same result.*

If the position of the prism in the figure be that in which the deviation is a minimum, then, since the angles of incidence and emergence are equal, the first angle of refraction, and the second angle of incidence, will also be equal; hence the triangle SII' will be isosceles, SP perpendicular to II' will bisect the angle ISI'; and denoting as before the refracting angle of the prism by G, and the first angle of refraction by R, we shall have $R = \frac{1}{2} G$.

*See note to this article at the end of the chapter.

Now, through O let the lines OB and OB′ be drawn parallel respectively to SA and SA′; then will

$$L'OI' = 180° - L'OB - BOB' - B'OE;$$

but $$L'OB = B'OE = LIA = 90° - LIN.$$

Hence, $$L'OI' = 180° - 180° + 2\,LIN - BOB';$$

or, calling the angle of incidence I, and the angle of minimum deviation D,

$$D = 2I - G,$$

and $$I = \frac{D + G}{2}.$$

Now, denoting the index of refraction by n, we have

$$\frac{\sin I}{\sin R} = n;$$

and hence, by substitution,

$$n = \frac{\sin \frac{D + G}{2}}{\sin \frac{G}{2}}.$$

To determine, therefore, the indices of refraction, we have only to measure the angles D and G.

49. *Determination of the indices of refraction.*

§ 1. *Solids.* The substance of which the index is sought, being wrought into the prismatic form, to measure the refracting angle ASA′ [Fig. 39′, Plate IV], we fix the prism in a position such that its edges shall be perpendicular to the plane of a graduated circle; and with a telescope which revolves

about an axis at O the centre of the circle, we view some distant object by a beam of light $S''IO$ reflected from the surface SA of the prism, and also by a direct beam $S'O$, and then read off on the graduated arc the angle $S'OI$ through which the telescope revolves in passing from the one position to the other. Removing the circle to O', we measure in a similar manner, by viewing the same object, the angle $S^{iv}O'I'$. The angle sought is equal to $\frac{1}{2}(S'IO + S^{iv}O'I')$. For, supposing the object so distant that the lines $S'O$, $S''I$, $S'''I'$, $S^{iv}O'$ may be regarded as parallel, and drawing SD parallel to their common direction, we have

$$ASD = SIS''.$$

But since $SIS'' = AIO$, we have

$$2\,SIS'' = 180° - S''IO = 180° - (180° - S'OI) = S'OI;$$

and hence $$ASD = \tfrac{1}{2}\,S'OI.$$

In the same manner it may be shown that

$$A'SD = \tfrac{1}{2}\,S^{IV}O'I'.$$

Hence we have

$$ASA' = \tfrac{1}{2}(S'OI + S^{IV}O'I').$$

To determine the minimum deviation, the prism is placed, with its axis vertical, on a movable horizontal stand, and in such a position that some very distant object may be viewed through it. Then at the distance of some feet, a repeating circle is adjusted so that its limb may be horizontal, and of the same height as the prism and the object; and

one of the telescopes is directed to some point of the object, and the other to the refracted image of it. The prism being made to revolve by means of the horizontal stand, and the second telescope being moved so as to keep in view the object as seen through the prism, the position of minimum deviation is soon found. The angle made by the telescopes is evidently the measure of the deviation.

§ 2. *Liquids.* The liquids are enclosed in hollow prisms, the faces of which are plane glasses.* The refraction observed is due entirely to the liquid; since by article 45 the direction of the emergent ray is the same as if the light had passed through the liquid only, without the interposition of the glass. The values of D and G are determined as in the case of solids.

§ 3. *Gases.* The same method is applicable to gases; but, as in them the refraction is much less than in liquids under the same circumstances, it is necessary that the prisms in which they are confined should have large refracting angles.

By employing a prism exhausted of air, the index of refraction for the passage of light from air into a vacuum, and the converse, may be determined.

By the preceding methods are obtained the indices of refraction, when the light passes from air into the various substances submitted to experiment. It remains to deduce from them the *absolute* indices;

* By a plane glass, is meant a glass with parallel plane faces.

that is, the indices when the light passes from a vacuum into these substances.

Referring to article 45, figure 39, let the index of refraction for light passing from the first medium into the second, be denoted by n'; from the first into the third, by n''; and from the second into the third, by x: then

$$\frac{\sin \mathrm{LIN}}{\sin \mathrm{N'II'}} = n', \quad \frac{\sin \mathrm{II'N''}}{\sin \mathrm{N'''I'I''}} = x, \quad \frac{\sin \mathrm{I'I''N^{IV}}}{\sin \mathrm{N^{V}I''L'}} = \frac{1}{n''};$$

and multiplying these equations term by term, and cancelling the equal angles, we have

$$1 = \frac{n'}{n''}x,$$

and

$$x = \frac{n''}{n'}.$$

That is, *the index of refraction for light passing from the second medium into the third, is equal to the quotient arising from dividing the index between the first and third, by that between the first and second.*

To illustrate the application of this principle to the case in question, let the index of refraction for the passage of light from a vacuum into water be required; the indices between air and a vacuum, and air and water, being known. Then, in the preceding enunciation, we have only to consider the first medium air, the second a vacuum, and the third water, and the index sought will be given by the equation

$$x = \frac{n''}{n'}.$$

By the same equation, we can determine the relative index of two substances, when their absolute indices are given. Thus, to find the index for the case of light passing from water into glass, the absolute indices of these substances being known, we have only to consider the first medium a vacuum, the second water, and the third glass.

The absolute indices in some of the most useful and interesting cases, are given in the following table:

Substances.	Indices.	Substances.	Indices.
Vacuum	1.000000	Chromate of lead from	2.50 to 2.97
Air	1.000294	Diamond from	2.47 to 2.75
Oxygen	1.000272	Phosphorus	2.224
Hydrogeu	1.000138	Native sulphur	2.115
Nitrogen	1.000300	Carbonate of lead from	1.81 to 2.08
Chlorine	1.000772	Ruby	1.779
Carbonic acid	1.000449	Tourmaline	1.668
Cyanogen	1.000834	Emerald	1.585
Ammonia	1.000385	Flint glass from	1.57 to 1.58
Sulphuret of carbon	1.678	Quartz	1.547
Oil of olives	1.470	Rock salt	1.545
Oil of turpentine	1.470	Canada balsam	1.532
Sulphuric acid, den'y 1.7	1.429	Crown glass	1.500
Nitric acid, density 1.48.	1.410	Borax	1.475
Hydrochloric acid, con'd	1.410	Alum	1.457
Alcohol	1.372	Fluor spar	1.436
Sulphuric ether	1.358	Ice	1.310
Water	1.336		

50. *Refraction by spherical lenses. A spherical lens* is a refracting medium included between portions of two spherical surfaces.

Let CC' [Fig. 40] be any straight line; and with C and C' as centres, let the arcs OAO', $OA'O'$ be

described, intersecting each other in O and O′; then if the figure OAO′A′ be made to revolve about CC′ as an axis, it will evidently generate a solid having the form of a lens, C and C′ being the centres of its surfaces or the *centres of curvature*, CA and C′A′ the *radii of curvature*.

The straight line CC′, indefinitely produced, is called the *axis of the lens*. The figure OAO′A′ is called the *profile* of the lens, and may evidently be considered a section of the lens by any plane passing through the axis.

In what follows, we shall suppose the luminous point to be situated on the right of the lens; OAO′ being its anterior, and OA′O′ its posterior surface.

AC $= r$, the radius of the anterior surface will be considered positive when it lies on the left of the point A, and negative when it lies on the right of A. A′C′ $= r'$, the radius of the posterior surface will be considered positive when it lies on the right of A′, and negative when on the left of that point.

By giving to r and r' suitable values and algebraic signs, all the possible forms of spherical lenses may be determined.

The substance of which lenses are usually constructed, is glass. Certain other transparent bodies are occasionally employed; but in all cases, the materials are such as possess a greater refractive energy than atmospheric air. If, then, A and B

[Figs. 41 and 44] be two lenses in which the convexities of the surfaces lie in opposite directions, and a beam of light fall upon them parallel to their axes, it is evident that in the one case the rays will be bent towards the axis, in the other from it*: from this circumstance, lenses are divided into the two classes of *converging* and *diverging lenses.*

§ 1. *Converging lenses.*

‡ 1. *The double convex lens.* [Fig. 41.]

This lens has both its surfaces convex. When the radii of the surfaces are equal, it is said to be equally convex. It is characterized algebraically by the equations

$$r = + a, \quad \text{and } r' = + a';$$

a and a' being any two finite quantities whatever.

‡ 2. *The plano-convex lens.* [Fig. 42.]

It has one surface plane, the other convex.

If its anterior surface is plane, it is characterized by the equations

$$r = \infty, \quad r' = + a'.$$

If its posterior surface is plane, the equations are

$$r = + a, \quad r' = \infty.$$

‡ 3. The *converging meniscus*, or *concavo-convex lens.* [Fig. 43.]

* The student will observe, that a ray of light is refracted by a lens in the same manner that it would be by a prism having its faces tangent to the lens at the points of incidence and emergence; that in the one case the base of the prism is directed towards the axis, in the other from it; and that rays parallel to the axis are always refracted towards the base of the prism.

In this lens one surface is concave, the other convex; *the radius of the convex surface being the smaller.*

If the anterior surface is convex, the equations are

$$r = + a, \quad r' = - a', \quad \text{and } a < a'.$$

If the anterior surface is concave, the equations are

$$r = - a, \quad r' = + a', \quad \text{and } a' < a.$$

§ 2. *Diverging lenses.*

‡ 1. The *double concave lens.* [Fig. 44.]

This has both surfaces concave, and is characterized by the equations

$$r = - a, \; r' = - a'.$$

‡ 2. The *plano-concave lens.* [Fig. 45.]

This has one surface plane, the other concave. If the anterior surface is plane, we have

$$r = \infty, \; r' = - a'.$$

If the posterior surface is plane, we have

$$r = - a, \; r' - \infty.$$

‡ 3. The *diverging meniscus.* [Fig. 46.]

In this lens one surface is concave, the other convex; *the radius of the concave surface being the smaller.*

If the anterior surface is convex, we have

$$r = + a, \quad r' = - a', \quad \text{and } a' < a.$$

If the anterior surface is concave, we have

$$r = - a, \quad r' = + a', \quad \text{and } a < a'.$$

51. In the formation of images by lenses, it is found to be essential to the distinctness of the image,

that the rays should fall upon the lens, and emerge from it *nearly at right angles.* Hence it follows:

1st. That the surfaces should be very slightly curved, so that in seeking approximate results, the arcs of the profile may be considered as *straight lines perpendicular to the axis.*

2d. That the rays, both incident and emergent, should make *very small angles* with the axis.

52. We now proceed to consider the refraction of a beam or pencil of light by spherical lenses.

Let OAO′A′ [Fig. 47] be the profile of a double convex lens, CC′ its axis, and C and C′ the centres of its anterior and posterior surfaces respectively.

We shall first suppose the luminous point to be situated somewhere *on the axis.*

Let L, then, be the position of the point, and LI a ray emanating from it. The ray, remaining constantly in the plane of the profile, will pass through the lens, and emerge from it in certain directions II′, I′P, and will intersect the axis in some point P; II′ being bent towards the radius CI, and I′P from the radius C′I′ produced.

As before, let

$$\mathrm{CA} = r, \quad \text{and } \mathrm{C'A'} = r';$$

also let

$$\mathrm{AL} = p, \quad \text{and } \mathrm{A'P} = q;$$

and denote the index of refraction for the passage of light from air into glass, by n.

It is required to find an equation which shall express approximately the mutual relations of these quantities.

The sine of a very small angle being sensibly equal to the angle itself, we have

$$\frac{HIL}{I'IM} = n, \text{ and } \frac{H'I'P}{II'M} = n;$$

or, $\quad HIL = n \,.\, I'IM, \quad$ and $H'I'P = n \,.\, II'M$;

and hence

$$HIL + H'I'P = n\,(I'IM + II'M),$$

or

$$\frac{HIL + H'I'P}{I'IM + II'M} = n.$$

But designating the acute angles made by the intersection of the axis with the radii of curvature and the incident and emergent rays by the letters at their vertices, viz. C, C′, L, and P, we have

$$HIL = C + L, \quad H'I'P = C' + P,$$

and

$$I'IM + II'M = I'MC = C + C'.$$

Hence, by substitution, we get

$$\frac{C + L + C' + P}{C + C'} = n,$$

and hence we have

$$\frac{C + L + C' + P}{C + C'} - 1 = n - 1;$$

or, reducing, $\quad \dfrac{L + P}{C + C'} = n - 1,$

or

$$L + P = (n - 1)\,C + (n - 1)\,C'.$$

Now since the angles P and L, C and C′ are very

small, we may employ in place of their tangents the angles themselves : we thus get

$$L = \frac{IA}{AL}, \quad P = \frac{I'A'}{A'P},$$

$$C = \frac{IA}{AC}, \quad C' = \frac{I'A'}{A'C'};$$

and hence, by substitution, we have

$$\frac{IA}{AL} + \frac{I'A'}{A'P} = (n-1)\frac{IA}{AC} + (n-1)\frac{I'A'}{A'C'}.$$

But since LI falls almost perpendicularly upon the lens, II′ is very nearly parallel to the axis; it is, also, on account of the thinness of the lens, very small : hence AI and A′I′ may be regarded as equal to each other, and the above equation will become

$$\frac{1}{AL} + \frac{1}{A'P} = (n-1)\frac{1}{AC} + (n-1)\frac{1}{A'C'},$$

or

$$\frac{1}{p} + \frac{1}{q} = \frac{n-1}{r} + \frac{n-1}{r'} \quad \ldots\ldots\ldots\ldots\ldots [\mathrm{D}]$$

The arcs AI and A′I′ do not enter into this equation; and the determination of q depends only upon p the distance of the luminous point, and upon the power of the lens given by the quantities r, r' and n. Thus, for a given lens, q is determined by p alone; and consequently all rays emanating from L, and making very small angles with the axis, will, after refraction, intersect each other on the axis sensibly at the same point P.

L and P are called *conjugate foci.*

53. Resuming the equation

$$\frac{1}{p}+\frac{1}{q}=\frac{n-1}{r}+\frac{n-1}{r'},$$

if we suppose $p=\infty$, and denote the particular value which q then assumes by f, we shall have

$$\frac{1}{f}=\frac{n-1}{r}+\frac{n-1}{r'} \ldots\ldots\ldots\ldots [\mathrm{E}]$$

Since in this case the luminous point is at an infinite distance from the lens, the conjugate focus F [Fig. 48] thus determined, is the focus of rays parallel to the axis. It is called the *principal focus* of the lens, and the distance A′F or AF (neglecting the thickness of the lens) is called the *principal focal distance.*

If in equation [D] we substitute for the second member the equivalent term $\frac{1}{f}$, as given by equation [E], we shall have

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \ldots\ldots\ldots\ldots\ldots\ldots [\mathrm{F}]$$

To apply the above formulæ to a particular case, the values of n, r and r', for the case in question, must be substituted in equation [E]. The value of f will thus be determined, and must be substituted in equation [F]. Then, when the value of p is given, that of q can be found, and the converse; that is, when one of the conjugate foci is known, the position of the other can be determined.

54. *Applications of equation* [E].

$$\frac{1}{f}=\frac{n-1}{r}+\frac{n-1}{r'},$$

In this equation, $(n-1)$ is always positive; for, as has already been remarked, lenses are always constructed of matter which possesses a greater refractive energy than atmospheric air, and consequently n is always greater than unity.

§ 1. *Converging lenses.*

In the double convex and plano-convex lenses the values of r and r' are both positive, and consequently the value of f is positive.

In the converging meniscus the value of either r or r' is negative; still, such is the relation between these radii, with respect to magnitude and algebraic sign, that the expression $\frac{n-1}{r}+\frac{n-1}{r'}$, and hence the value of f, is always positive.

In converging lenses, therefore, f is always positive; and the principal focus is situated behind the lens, and is real.

Particular cases.

‡ 1. *The double convex lens.*

Here $r=+a$, and $r'=+a'$;

and we have

$$\frac{1}{f}=\frac{n-1}{a}+\frac{n-1}{a'},$$

and

$$f=\frac{aa'}{(n-1)\,a+(n-1)\,a'}.$$

If we suppose $n = \frac{3}{2}$, which is a near approximation for some kinds of glass, we shall have $(n - 1) = \frac{1}{2}$, and hence

$$f = \frac{2aa'}{a + a'}.$$

From this formula may be derived the following rule for finding the principal focal distance of a double convex lens :

Divide twice the product of the radii of the surfaces by the sum of the radii ; the quotient will be the distance sought.

If the lens is equally convex, $a = a'$,

and $$f = a;$$

that is, *the principal focal distance is equal to the common radius of the surfaces of the lens.*

‡ 2. *The plano-convex lens, the anterior surface being plane.*

Here $$r = \infty, \text{ and } r' = + a';$$

and equation [E] becomes

$$\frac{1}{f} = \frac{n - 1}{a'}.$$

If $$n = \tfrac{3}{2}, \quad f = 2a';$$

that is, *the principal focal distance is equal to twice the radius of the convex surface.*

If we suppose the posterior surface to be plane, we shall find

$$f = 2a.$$

Hence, if $a = a'$, that is, if the same lens be employed in both cases, the principal focal distance will be the same, whichever surface be directed to the incident beam.

§ 2. *Diverging lenses.*

If we examine the different cases, we shall find that in diverging lenses f is always negative; and hence that the principal focus is situated in front of the lens, and is virtual.

The application of equation [E] to particular cases, is so easy, that it may be left as an exercise for the student.

55. *Applications of equation* [F].

Resolving equation [F] with respect to q, we find

$$q = \frac{pf}{p-f} = \frac{f}{1-\frac{f}{p}} \quad \text{............[G]}$$

§ 1. *The double convex lens.*

Equation [G] is immediately applicable to the present case; and we have

$$q = \frac{f}{1-\frac{f}{p}}.$$

In discussing this equation, we shall consider the luminous point as moving from a position at an infinite distance from the lens, describing that part of the axis which lies in front of it, and finally coinciding with the anterior surface.

‡ 1. $p > f$, and hence $\frac{f}{p} < 1$ and q positive.

When $p = \infty$, we have $q = f$; that is, when the luminous point is at an infinite distance, the image is formed at the principal focus F. [Fig. 49.]*†

As p diminishes, $\frac{f}{p}$ increases and $(1 - \frac{f}{p})$ diminishes, and hence q increases; that is, as the luminous point moves from its position at an infinite distance, *towards* the lens, the image moves from its position at the principal focus F, *from* the lens.

If we suppose $p = 2f$, we have $q = 2f$; that is, the luminous point and its image arrive at twice the focal distance from the lens at the same time; while therefore the luminous point advances from infinity to the point G, the distance of which from the lens is equal to $2f$, the image recedes from the principal focus F only through a distance FH equal to f.

‡ 2. $p = f$, and hence $\frac{f}{p} = 1$.

This supposition renders q infinite; hence, while the luminous point advances from G to L, or from

*This result, it will be observed, is a necessary consequence of the definition of the focus.

†Let $p = 1000f$, then $q = f + \frac{f}{999}$.

If, for example, f is equal to one inch, the image is formed at a distance from the principal focus equal to only the $\frac{1}{999}$ of an inch. Thus the image is formed sensibly at the principal focus when the object is at a distance, which, like $1000f$, may be considered great with respect to f.

the distance $2f$ to the distance f from the lens, the image recedes from H, or the distance $2f$, to infinity.

‡ 3. $p < f$, and hence $\frac{f}{p} > 1$ and q *negative.*

The negative sign with which q is affected, shows that the image is formed in front of the lens, and is consequently virtual.

As p diminishes, $\frac{f}{p}$ increases, $(1 - \frac{f}{p})$ also increases, and q diminishes; and when $p = 0$, we have $q = 0$; that is, while the luminous point moves from L to A or A′ (neglecting the thickness of the lens), the image moves from an infinite distance in front of the lens to the same point.

In the preceding articles we have supposed the light to emanate from a luminous point placed in front of the lens, and consequently the rays to be either diverging or parallel; but it is sometimes necessary to consider rays as converging to a point behind the lens, as, for instance, when parallel rays having been brought to a focus by a converging lens, we interpose a second lens between the focus and the first lens.

To adapt equation [G] to such a case, it is evidently necessary only to make p negative.

§ 2. *The double concave lens.*

In this case f is negative, and equation [G] becomes

$$q = -\frac{f}{1 + \frac{f}{p}}.$$

Here for every positive value of p, q is negative; hence, at whatever point on the axis the luminous point may be placed in front of the lens, the image is also in front of it, and virtual.

If we suppose $p = \infty$, we have $q = -f$; as p diminishes, $1 + \frac{f}{p}$ increases and q diminishes; and when $p = 0$, we have also $q = 0$; that is, as the luminous point moves from a position at an infinite distance from the lens to the anterior surface, the image moves only from the principal focus F [Fig. 50] to the same surface.

56. When the pencil of light emanates from a point situated on the axis of the lens, one of its rays coincides with the axis, and passes through the lens without refraction; this ray is called the *axis of the pencil.*

Since we are now to consider a pencil emanating from a point placed a little without the axis of the lens, our first step must be to determine what we are to regard as the axis of such a pencil. For this purpose, a point called the *optic centre* of the lens must be defined, and its properties made known.

C and C′ being the centres of curvature of the lenses, represented in figures 51, 52, 53 and 54, let CN and C′M, any two parallel radii, be drawn; and also through the points M and N, let a straight line be drawn, and produced, if necessary, to intersect the axis; O, the point of intersection, is called the

optic centre. It is to be shown, that in the same lens this point is invariable.

The triangles CNO, C′MO give

$$CO : C'O :: CN : C'M;$$

and for the lenses in figures 51 and 52, we have

$$CO + C'O : CN + C'M :: CO : CN,$$

or $$CC' : CN + C'M :: CO : CN;$$

and hence, $$CO = \frac{CN \times CC'}{CN + C'M} \ldots\ldots\ldots\ldots\ldots\ldots [m]$$

For the lenses in figures 53 and 54, we have

$$CO : C'O - CO :: CN : C'M - CN,$$

or $$CO : CC' :: CN : C'M - CN,$$

and hence, $$CO = \frac{CN \times CC'}{C'M - CN} \ldots\ldots\ldots\ldots\ldots\ldots [n]$$

The quantities CC′, CN and C′M, in the second members of equations [*m*] and [*n*] being constant for the same lens, CO is also constant; but CO determines the point O; hence, in the same lens, the position of the optic centre is *invariable.*

Now, the tangents, at the points M and N, are evidently parallel to each other; hence, if a ray of light LN fall upon the surface of the lens at N, at such an angle as, after refraction, to take the direction NM, it will emerge in a direction ML′ parallel to LN; and if through O we draw SS′ parallel to LN, we may, on account of the thinness of the lens, suppose it to coincide with the refracted ray LNML′. We may, therefore, say that a ray which passes through the optic centre of a lens suffers no sensible refraction.

In the case, therefore, of a pencil of light emanating from a point situated without the axis of the lens, the axis of the pencil is the ray which passes through the optic centre.

Now, let the luminous point L′ [Fig. 55] be a little without the axis, and through the optic centre O draw the straight line L′OP′; let L′I be any ray emanating from L′, making a very small angle with the axis of the lens, and after refraction meeting the ray L′OP′ in P′: *required the relation between* OL′ *and* OP′, *the distances, respectively, of the point and conjugate focus from the lens.*

Designating the acute angles by the letters at their vertices, viz. L, P, L′, P′, and neglecting the refraction at I′, we have

$$P + L = P' + L'.$$

Since the point L′ is very near the axis, the angle L′OL is very small, and IBL′, IBP′ may be considered right angles. Also, since L′I is nearly parallel to the axis, the acute angles P, L, P′, and L′, are very small; hence, using the angles in place of their tangents, we have

$$P = \frac{IA}{AP}, \quad L = \frac{IA}{AL};$$

$$P' = \frac{IB}{BP'}, \quad L' = \frac{IB}{BL'}.$$

These values substituted in the above equation, give

$$\frac{IA}{AP} + \frac{IA}{AL} = \frac{IB}{BP'} + \frac{IB}{BL'};$$

or, neglecting the small difference AB,

$$\frac{1}{AP}+\frac{1}{AL}=\frac{1}{BP'}+\frac{1}{BL'}.$$

It is evidently a matter of indifference, so far as the refracted ray IP is concerned, whether we suppose the light to emanate from L′ or L. Let it be supposed to emanate from L: P will then be the corresponding conjugate focus; and if the principal focal distance be denoted, as before, by f, we shall have [Equa. F],

$$\frac{1}{AL}+\frac{1}{AP}=\frac{1}{f};$$

and hence,

$$\frac{1}{BP'}+\frac{1}{BL'}=\frac{1}{f};$$

or, denoting the distances BL′ and BP′ by p' and q' respectively,

$$\frac{1}{p'}+\frac{1}{q'}=\frac{1}{f}\text{..................}[\text{H}]$$

From this equation, it appears that when the luminous point is a *little without the axis*, as well as in the case in which it is on the axis, all the rays that fall upon the lens at small obliquities, meet after refraction sensibly in the same point.

The discussion of equation [H] is evidently similar to that of equation [F].

57. *The formation of images by lenses.*

The formation of images by lenses can be readily explained by the foregoing principles. The object placed before the lens, and at such a distance, or of

such dimensions, as to answer the requisite conditions, sends forth from each of its points diverging rays, which, after refraction, intersect the axis of the pencil at a distance from the lens, determined by equation [H], and form an image of the point. Thus are formed images of all the points of the body, constituting by their union the *image of the body*.

The subject may be illustrated by the following geometrical constructions.

Let OI [Fig. 56] be a converging lens, NM its axis, and F its principal focus; and let the object be a straight line perpendicular to the axis, and bisected by it.

§ 1. Let the object LQ be at M, OM being equal to 2 OF; that is, twice the focal distance.

Of the rays which emanate from the top of the object, the ray LI, parallel to the axis, is so refracted as to pass through the principal focus F, and meet the ray LO, which passes through the optic centre in some point S. SR, drawn perpendicular to the axis and bisected by it, is the image of LQ.

Since IL = 2 FO, we have LS = 2 SO, and LO = OS, and hence the triangles OML, ONS equal; consequently the image, inverted, is equal to the object, and at the same distance from the lens.

§ 2. Let the object be at M′, some point between M and M″, OM″ being equal to OF; then, the ray L′O, drawn through the optic centre, will evidently make a greater angle with the axis than in the pre-

ceding case, and will meet the ray IF produced in some point S′, more distant from the lens than S. The image, still inverted, will, therefore, be at a greater distance than the object, and magnified.

§ 3. Let the object be at M″; then, L″O and IF will be parallel; and the image will be at an infinite distance.

Thus, while the object moves from M to M″, the image moves from N to an infinite distance.

§ 4. Let the object be at M‴, a point within the principal focal distance. Here, the ray L‴O will evidently meet the ray IF produced, in some point S‴ in front of the lens, and the image will be virtual. It is also clear that the image will always be erect and magnified; and that its distance from the lens will be the greater, the nearer the object to the point M″.

§ 5. Let the object be at M^{iv}, at a distance greater than 2OF. Then will the ray $L^{iv}O$ make a less angle with the axis than LO, and meet the ray IF produced, in some point S^{iv}, nearer the lens than the point S. As the distance of the object from the lens increases, the angle $L^{iv}OM^{iv}$ diminishes; and when the distance becomes infinite, is reduced to zero: the ray then coincides with the axis, and intersects IF in the point F. Thus, while the object moves from M to an infinite distance, the image, constantly diminishing, and inverted, moves only from N to F.

Similar constructions are applicable to diverging lenses.

The above results are verified by experiment.

Before dismissing the subject of optical images, it may be proper to direct the attention of the student to an essential difference, in respect to visibility, between the object and its image, viz. that while from the object light emanates in all directions, from the image it emanates only in certain directions; so that, while the object is visible in all positions of the eye, provided an opaque body be not interposed between them, the image is visible only when the eye is situated somewhere within the limits of the converging or diverging pencils of rays by which it is formed.

The principal focal distance of lenses, whether converging or diverging, may be determined experimentally by processes similar to those employed in the case of mirrors.

58. *The spherical aberration of lenses.*

When the conditions of article 51 are satisfied, we have seen that the rays of a pencil, emanating from a luminous point placed either on the axis or a little without it, are brought to a focus sensibly at the same point.

But when these conditions are not fulfilled, it is evident, that while the rays incident upon the central part of the lens converge to a focus at some point F [Fig. 57], those incident upon an annulus

near the edge of the lens will converge to some point F less distant from the lens than F, and that thus will result an aberration similar to that described in article 34. It is called the *spherical aberration of the lens*, the distance FF′ being denominated the *longitudinal*, MN the *lateral aberration.*

The effect of spherical aberration is to prevent the formation of a clear and well defined image. To correct it, when the central part is of sufficient extent for the purpose for which the lens is required, it is only necessary to restrict the incident light to it. This is effected by placing before the lens an opaque screen, called a *diaphragm*, having a circular opening of such a size as to admit no rays but those which converge nearly enough to the same point. The diameter of the opening determines the *aperture* of the lens.

But this method is not always eligible, since, in diminishing the aberration, it diminishes also the light, and consequently the brilliancy of the image. Hence the necessity of some means of counteracting the aberration, which shall not lessen the aperture of the lens.

In the case of a single lens, the aberration may be very much reduced by giving certain curvatures to its surfaces. Thus, while in a double convex lens equally convex, made of glass, for which n the index of refraction is supposed equal to 1.5, the longitudinal aberration for parallel rays is equal to 1.67 of

its thickness; in a double convex lens of the same kind of glass, the radii of which are as 1 to 6, the aberration for parallel rays, incident upon the face whose radius is 1, is only 1.07. The latter is the *lens* which, with a given power, has the least aberration for parallel rays. It is called by opticians the "*crossed lens.*"

In the case of two lenses in contact, such magnitudes and directions may be given to the curvatures of their surfaces, that, for parallel rays at least, the partial aberrations will completely compensate each other. Combinations may thus be obtained entirely free from aberration, and hence called *aplanatic.*

59. It can be demonstrated, that lenses in which one of the surfaces is spherical and the other elliptical or hyperbolic, described under certain conditions, will cause parallel rays to converge accurately to a single point. But the practical difficulties in the construction of such lenses have hitherto proved insuperable.

60. When the refracted rays do not converge to a single point, they form, by their successive intersections, a curved surface, called a *diacaustic*, exactly analogous to the catacaustic produced by the intersections of reflected rays. Figure 58 represents a section of the caustic, formed by the refraction of parallel rays by a double convex lens; figure 59, a similar section, the lens being double concave.

NOTES.

NOTE TO ARTICLE 46.

FIGURE 34 is so constructed, that the effect of the refraction is to bend the ray from the vertex of the prism, and towards its base.

This may be shown to be true in all cases in which the prism is composed of matter more refractive than the surrounding medium.

The different cases are presented in figures 60, 61, 62, and 63.

In figures 60 and 61, SIL being an obtuse angle, LI produced falls between S and the perpendicular IN′; and hence the ray II′ is bent from S. If the angle II′S is acute, as in figure 60, the ray is still bent from S: if obtuse, as in figure 61, it is bent towards S.

In figure 62, where NIS is a right angle, there is no refraction at I; and NI′S being an acute angle, it is clear that the ray is bent from S.

In figure 63, in which LIS is acute, II′ is bent towards S at I, and from it at I′.

The 2d and 4th cases [Figs. 61 and 63] are, then, the only ones that require farther examination.

But in figure 61, if we suppose the ray to pass, both at I and I′, from the prism into the surrounding medium, the angles N′II′, NI′I become the angles of incidence, and NIL and N″I′E the corresponding angles of refraction. Admitting, then, the truth of a principle which will be immediately demonstrated, viz. that the greater the angle I of incidence, the greater will be the angle (I — R) of *deflection;* and observing that the angle N′II′ is greater than NI′I, it becomes evident that the inflection of the ray IL, from the vertex S, is greater than the inflection of the ray I′E towards S; and, consequently, that the joint effect of the refractions at I and I′ is a bending of the incident ray towards the base of the prism.

In figure 63, since $N''I'I$ is greater than $N'II'$, it is also evident that the ray is in like manner bent by the refraction towards the base.

To show that the angle of deflection increases with that of incidence, let it be denoted by D'; then,

$$D' = I - R:$$

we also have $$\sin R = \frac{1}{n} \sin I;$$

and hence, by substitution,

$$D' = I - \sin^{-1}\left(\frac{1}{n} \sin I\right).$$

Considering D' as a function of I, and differentiating, we get

$$\frac{dD'}{dI} = 1 - \frac{\frac{1}{n} \cos I}{\sqrt{1 - \frac{1}{n^2} \sin^2 I}};$$

and substituting the value of n.

$$\frac{dD'}{dI} = 1 - \frac{\frac{\sin R \cdot \cos I}{\sin I}}{\cos R}$$

$$= 1 - \frac{\tan R}{\tan I}$$

$$= \frac{\tan I - \tan R}{\tan I}.$$

Now, if $n > 1$, we have $I > R$,

and D' and $\frac{dD'}{dI}$ are both positive;

if $n < 1$, we have $I < R$,

and D' and $\frac{dD'}{dI}$ are both negative.

Thus, D' and its differential coefficient have always the same sign; and hence D' is always an increasing function of I.

NOTE TO ARTICLE 48.

Denoting the angles EI′N‴, II′K [Fig. 34] by I′ and R′ respectively, and representing by the letters I and R, D and G, the same angles as before, we have

$$\text{MO}'\text{E} = \text{O}'\text{II}' + \text{O}'\text{I}'\text{I},$$

or

$$\text{D} = \text{I} - \text{R} + \text{I}' - \text{R}'.$$

We also have

$$\text{R} + \text{R}' = \text{G};$$

hence,

$$\text{D} = \text{I} + \text{I}' - \text{G}.$$

But

$$\sin \text{I} = n \sin \text{R},$$

or

$$\text{I} = \sin^{-1}(n \sin \text{R});$$

and

$$\sin \text{I}' = n \sin \text{R}',$$

or

$$\text{I}' = \sin^{-1}(n \sin \text{R}') = \sin^{-1}(n \sin (\text{G} - \text{R})).$$

Hence, by substitution,

$$\text{D} = \sin^{-1}(n \sin \text{R}) + \sin^{-1}(n \sin (\text{G} - \text{R})) - \text{G}.$$

Thus D is a function of R alone. To determine the minimum value of which we know it to be susceptible, we have, by differentiation,

$$\frac{d\text{D}}{d\text{R}} = \frac{n \cos \text{R}}{\sqrt{1 - n^2 \sin^2 \text{R}}} - \frac{n \cos (\text{G} - \text{R})}{\sqrt{1 - n^2 \sin^2 (\text{G} - \text{R})}} = 0.$$

Transposing, squaring, and substituting for $\cos^2$ its equal $1 - \sin^2$, we get

$$\frac{1 - \sin^2 \text{R}}{1 - n^2 \sin^2 \text{R}} = \frac{1 - \sin^2 (\text{G} - \text{R})}{1 - n^2 \sin^2 (\text{G} - \text{R})}.$$

Reducing, we find

$$(n^2 - 1) \sin^2 \text{R} = (n^2 - 1) \sin^2 (\text{G} - \text{R}),$$

or

$$\sin^2 \text{R} = \sin^2 (\text{G} - \text{R}),$$

or

$$\text{R} = \text{G} - \text{R},$$

and

$$\text{R} = \tfrac{1}{2}\text{G};$$

hence

$$\text{R}' = \tfrac{1}{2}\text{G}.$$

R and R′ are therefore equal, and consequently I and I′ are equal. That is, the deviation is a minimum when the angles of incidence and emergence are equal.

CHAPTER III.

OF THE DECOMPOSITION OF LIGHT.

61. It has already been remarked that light, in undergoing refraction by a prism, appears to be resolved into rays of different colors. To this phenomenon the attention of the student is now to be directed.

If a beam LL′ [Fig. 64] of solar light be admitted into a dark chamber by a very small aperture M, and received on a screen SS′, a circular image O of the sun will be formed on the screen, of greater or less diameter according to its distance from M. But if we interpose a glass prism, having its edge horizontal, and also perpendicular to the direction of the beam, so as to intercept the rays near the aperture, instead of the circular colorless image O, there will be formed a colored and elongated image RV. This image is called the *solar spectrum*, and is represented in figure 65. It is terminated laterally by vertical straight lines, and at the ends by semicircles. Its breadth is always equal to the diameter of the direct image O; its length varies with the refracting angle of the prism, and the substance of which it is made.

Commencing with that end of the spectrum which is formed by the rays that are least refracted, the colors succeed each other in the following order, viz: *red, orange, yellow, green, blue, indigo, violet.*

It must not be imagined, however, that each of these spaces is of a uniform tint throughout. On the contrary, each space comprehends an endless variety of shades, and the transition from one principal color to another is made by gradations so nearly imperceptible that it is difficult to fix upon a line of separation between them. The following table exhibits the relative lengths of these spaces, as determined by Fraunhofer, with a prism of flint glass.

Red, - - - - -	56
Orange, - - - -	27
Yellow, - - - -	27
Green, - - - -	46
Blue, - - - - -	48
Indigo, - - - -	47
Violet, - - - - -	109
Total length, - -	360

An immediate inference from the preceding experiment is, that *a beam of white light consists of an almost infinite variety of rays, differing from each other in color and refrangibility.* For as the violet rays, for instance, emerge from the prism at a greater angle than the red rays, although the angle of incidence and the refracting medium are the same for each, we must admit that the former are more

refrangible than the latter. For a similar reason, we must conclude that rays of any two different tints of the same principal color have different refrangibilities; thus, for example, that the extreme violet are more refrangible than the mean violet rays.

Admitting the above principle, the solar spectrum may be conceived to be made up of a series of superimposed circular images, having their centres on the same line at right angles to the edge of the prism. Thus, the extreme red rays may be supposed to form a red image of the sun of the tint peculiar to them; the red rays, next in the order of refrangibility, to form a second image of a little different tint, not entirely superimposed upon the first, but a very little advanced towards the violet end, and so on throughout the spectrum.

According to the above explanation, the spectrum should be terminated at its extremities by semicircles; and the lines which bound it laterally, being made up of indefinitely small arcs of circles, ought to be sensibly straight: both which circumstances occur, as we have seen, in the spectrum.

62. If, in the preceding experiment, the screen SS′ [Fig. 64] be perforated opposite any of the colored spaces, the violet, for example, a small beam of violet light may be obtained and examined separately. When this beam is intercepted by another prism *abc*, and, after refraction at *f* and *g*, is received on

a second screen S″S‴, it is found not to produce a colored spectrum like the original beam of which it formed a part, but simply a luminous space of the same tint as the incident pencil V*f*.

The beam *gh* may, in like manner, be again and again refracted by prisms or lenses, but it will undergo no further change. Hence, it appears that the light which proceeds to any particular point of the spectrum, is incapable of farther separation into rays of different colors by subsequent refractions.

From this circumstance, the seven principal colors of the spectrum are called *primary colors*. Combinations of these primary colors are called *secondary colors*.

63. In the foregoing articles, it has been shown that a beam of white light can be separated by refraction into certain elementary rays, or decomposed. We have now to show that white light can be reproduced by compounding these same rays.

§ 1. *By rendering the elementary rays parallel to each other.*

This can easily be effected by means of two prisms of the same kind of glass, and of equal refracting angles. If such prisms ABC, DFE [Fig. 66] are placed very near each other, with their refracting angles in opposite directions, a beam of light incident upon the surface AB of the first prism will emerge from the surface EF of the second, parallel to its primitive direction, and colorless.

In this experiment, the elementary rays into which the incident beam is separated by the first prism, emerge from the surface AC, and pass through the thin lamina of air ACED, divergent and colored; but the refractions which they suffer in the second prism exactly compensate those of the first, and hence they emerge from the surface EF parallel to each other. Since, then, the experiment shows that they also emerge colorless, we conclude that the whiteness is due to the coincidence and blending of the elementary colored rays.

§ 2. *By causing the elementary rays to pass through the same point.*

If the spectrum RV [Fig. 67] be formed on a large concave mirror MM′, the image, received on a small screen placed at F the focus of the mirror, will be colorless. If the screen be at a little less distance from the mirror than the focal distance, the recomposition will be incomplete, and the extreme colors of the spectrum R′, V′, will appear in the same order as on the mirror; if at a little greater distance, the same colors will still appear, but in an inverted order V″, R″.

If we place at the focus a small and highly polished mirror M, and receive the light reflected from it upon a screen, the image V‴R‴ will yet be a spectrum, showing that the rays, though producing colorless light by their intersection at the focus, still preserve an independent existence.

§ 3. *By superposition.*

If a circular disk, divided into sectors, painted successively with the seven primary colors, and proportional in extent to the space occupied by these colors in the spectrum, be made to revolve rapidly, it will appear of a white color, more or less pure, according as the colors of the sectors bear a greater or less resemblance to the colors of the spectrum. To explain this, we must admit that the impression which each sector produces upon the eye, continues for a sensible, though a very short time; a principle familiarly illustrated in the production of a luminous curve by rapidly whirling an ignited coal. Hence, when a certain degree of rapidity is attained, each sector, considered by itself, will present to the eye the appearance of a complete circle of its own color; thus, the eye will perceive circles of all the primary colors at the same time; and since the total effect is to produce a white circle, we must admit that the primary colors, superimposed and intermingled, produce colorless light.

We have an illustration of this principle in the appearances presented by objects seen through a prism. When a rectangular strip of white paper, parallel to the axis of the prism and illuminated by solar light, is thus viewed, it exhibits at the upper and lower edges fringes of certain colors of the spectrum, but *the intervening part still appears white.* To explain this, we consider the rectangular slip as made up of a number of very narrow similar slips;

then, since each of these forms a complete spectrum, there will evidently be a superposition of spectra producing colorless light. At the upper and lower edges there will necessarily be an absence of some of the elementary rays, and hence these edges will appear colored.

64. *The formation of secondary colors by suppressing a part of the elementary rays.—Complementary colors.*

By receiving each of the primary colors on a separate moveable mirror, and giving suitable inclinations to the mirrors, the rays of any of the colors we may wish to compound, may be directed to the same point, and the resulting color be exhibited on a *white screen.* When the mirrors are so inclined as to reflect all the colors to the same point, the experiment is identical with that of Art. 63, § 2; and, as we should expect, white light is produced. When the violet light is suppressed, and the remaining colors compounded, the light produced is of a yellowish hue; when the blue and green are successively suppressed, this yellowish hue passes through orange to scarlet and blood red. When the red end of the spectrum is suppressed, and more and more of the less refrangible rays are successively abstracted, the light passes from pale green to vivid green, blue green, blue, and finally into violet. When the rays of the middle portion of the spectrum are withdrawn, various shades of purple, crimson, and plum color, result.

In this manner, by varying the suppressed rays, any color or shade of color, existing in nature, may be exactly imitated, with a brilliancy surpassing that of any artificial coloring.

If, in any of the above cases, we should unite the suppressed rays, it is evident that the resulting color, combined with that on the screen, would reproduce white light. Any two colors which, thus combined, produce white light, are called, with reference to each other, *complementary colors*.

65. *The color of bodies.*

In the preceding article we have seen that a *white screen* may be made to appear of the various shades of yellow, orange, scarlet, etc., by concentrating upon it the proper elementary rays.

When a *colored sereen* is used, the same general results are obtained, the screen appearing of the same color as the light thrown upon it. In the cases, however, in which its color is of the pure and intense class, as vermilion or prussian blue, the light exhibited on the screen is much the most vivid when it comes from the part of the spectrum that most nearly resembles the hue of the screen in white light. Thus, while the red rays thrown upon a vermilion-colored screen produce a most brilliant red, the yellow rays produce a yellow less bright than the yellow of the spectrum, the green rays a dull green, and in the case of the indigo and violet rays there is almost a complete absorption, the screen appearing nearly black.

If the above results are duly considered in connexion with the familiar fact that different substances exposed to white light exhibit different colors, it will be perceived that they warrant the following conclusions :

§ 1. That when a beam of white light is incident upon the surface of a body, it is in general decomposed; that certain of the elementary rays, constituting by their union the color of the body, are reflected or rather *radiated** from it, whilst the remainder are absorbed, or, if the body is transparent, are partly absorbed and partly transmitted.

§ 2. That in general these decompositions, radiations, etc., vary with the substance of the body; thus, that one body, in consequence of some peculiarity in its constitution, radiates certain rays, and absorbs, or transmits and absorbs others more copiously than another body of a different nature. That a body of a green color, for example, radiates only those elementary rays, which, combined, form the peculiar green tint which it exhibits, and absorbs, or transmits and absorbs the rest; that a white body radiates equally all the elementary rays, and that a body of a black color absorbs all or nearly all.

*We use the term *radiated* rather than *reflected*, because the color appears to be due not to a mere reflection, but to some action originated by the incident light in the body itself, by which it becomes a new source of light, a source of rays peculiar to itself.

The object of a theory of colors is to explain the above facts, to show in what manner these various radiations, absorptions and transmissions take place.

When the light which emanates from colored substances, whether of vegetable, mineral or animal origin, is examined by the prism, it is found to be compound. Thus, the petals of a flower, however closely resembling in hue any one of the primary colors of the solar spectrum, always exhibit several distinct shades when viewed through the prism.

66. *Results of the analysis of artificial light.*

Light of various kinds, in each of which some one color is generally predominant, can be produced by artificial means, as by the direct ignition of metallic vapors and certain gases, by dissolving metallic salts in alcohol and igniting the solution,* etc. The results of the analysis of the light thus produced are: 1st. That no simple color is found in it, which does not exist in solar light. 2d. That no spectrum can be formed by it in which the simple colors are seen in the relative intensities and proportions in which they appear in the solar spectrum, the color which predominates in the artificial light predominating also in its spectrum; a red flame, for instance, producing a spectrum in which the prevailing hue is red, a blue flame a spectrum in which it is blue.

*The flame of alcohol in which common salt has been dissolved, emits a yellow light, which is found to be almost homogeneous.

In the fact that artificial light is always deficient in some of the elements of solar light, we find an explanation of the well known phenomenon that the same body frequently appears of one color during the day, when seen by solar light, and of another at night, when seen by the light of a lamp or candle.

66′. *Intensity of the light at different parts of the spectrum.*

The most luminous part of the solar spectrum is at the middle of the yellow space; from which the intensity of the light gradually diminishes to the outer extremities of the red and violet spaces. The least luminous part is the violet space.

67. *The calorific effects of the solar rays.*

When very delicate thermometers are placed in the colored spaces of the solar spectrum, different degrees of temperature are indicated.* If prisms of different substances are employed, it is found that while the temperature in general increases from the violet to the red end of the spectrum, the point where the heat is at its maximum varies with the prism; being, when the prism is of water, in the yellow space; of sulphuric acid, in the orange; of crown or plate glass, at the middle of the red; of flint glass, beyond the red space; of rock salt, still further beyond it.

*Instead of the thermometer, a modification of the thermo-electric pile may be employed, an instrument by which almost infinitesimal variations of temperature are rendered apparent.

It is found that the change of position of the point of maximum temperature as the substance of the prism is changed, is due to the fact that a greater or less number of the calorific rays is absorbed by the prism, and that different kinds of matter absorb rays of different degrees of refrangibility unequally. Thus in a prism of water, the calorific rays having the same refrangibility, as the red and orange, are absorbed in greater numbers than those which correspond to the yellow rays, and the maximum occurs in the yellow space.

Rock salt being almost perfectly diathermanous, the spectrum formed by a prism of that substance may be regarded as the normal spectrum for calorific rays.

68. *The chemical effects of the solar rays.*

The solar rays produce chemical effects. Thus, on exposure to light, the chloride of silver is decomposed, chlorine and hydrogen enter into combination, fabrics in which the coloring matter is of organic origin fade, and plants which have been blanched by growing in the dark recover their green color. Light also produces certain physical effects involving a change in the arrangement of the molecules without change of chemical composition, as in the case of phosphorus, which, from being transparent and almost colorless, becomes, by exposure to light, opaque and of a red color.

To the chemical effects of the sun's rays we are indebted for the wonderful results of photography,

the fundamental process in which consists in subjecting certain highly sensitive chemical substances, very thinly spread over metallic or other surfaces, to the action of solar light.

The property of effecting chemical changes exists only in that part of the spectrum which is formed by the more refrangible rays, extending from the green to the extreme violet and even beyond the visible spectrum. The position of the point of maximum chemical effect varies with the nature of the substance exposed to the action of the rays.

The invisible rays which exist beyond the extreme violet may be rendered visible by receiving them on a converging lens, by which they will be collected into a faint beam of lavender-colored light.

When these invisible rays are received on certain substances, as, for example, on paper that has been washed in sulphate of quinine, they are changed into rays of less refrangibility and become visible, the color emitted being usually of a pale blue. This phenomenon is called *fluorescence.*

Among the more refrangible rays of the solar spectrum are rays which possess the property of producing *phosphorescence;* that is, of communicating to certain bodies, not luminous under ordinary circumstances, the power of emitting a faint light for a longer or shorter time after being withdrawn from the influence of the rays. These rays have been called *phosphorogenic* rays.

69. *The fixed lines of the spectrum.*

In order that the colors of the solar spectrum may be effectually separated from each other, a little consideration renders it evident that the aperture by which the light is admitted must be very small, and the screen at a certain distance from the prism. But these are not the only conditions. The sun has an apparent diameter of considerable magnitude (about 32′), and hence the solar images of the different colors overlap each other. This effect must be counteracted: a convenient method of doing it consists in forming a minute image of the sun by means of a convex lens of small focal length, and intercepting, at some distance, the light which emanates from it, by a screen pierced with a very small aperture. The light which passes through the aperture may be considered as emanating from a physical point.

Another cause of the imperfect separation of the different rays exists in the prism itself. Ordinary prisms are full of veins and striæ, by which the light is dispersed irregularly, and rays which belong to different parts of the spectrum are intermingled. Prisms are seldom found entirely free from these defects; and when they cannot be procured, hollow prisms, filled with water or any of the more dispersive oils, may be employed; or the chances of encountering the defects of a bad prism may, to some extent, be diminished by transmitting the rays as near the edge as possible.

When the above conditions are fulfilled, a spectrum is formed, greatly reduced in breadth, in which the different colors are no longer sensibly capable of farther separation; but this is not all: a most curious phenomenon is now observed. Instead of a continuous brilliant line, such as we should expect to see, in which the degrees of illumination glide imperceptibly into each other, we perceive a line in which there are numerous abrupt changes in the intensity of the light, producing, at some points, intervals almost black.

The precautions to obtain a more perfect separation of the rays, having reduced the spectrum to a mere line, to procure one of greater breadth, we change the minute circular aperture in the screen, through which the light passes, into a very narrow vertical aperture; then, giving to the prism, also, a vertical position, a spectrum is formed, horizontal and rectangular, consisting really of a great number of linear spectra, placed side by side.* The spectrum, as thus formed, appears to be crossed vertically by a multitude of dark lines, distributed irregularly throughout its whole extent. FRAUNHOFER, to whom this discovery is due, by receiving the spectrum on the object glass of a telescope, and thus magnifying the lines, ascertained the existence of nearly six hundred of them. From these he

*A cylindrical instead of a spherical lens may be employed, the former giving directly a brilliant *line* of light.

selected seven, on account of their distinctness and the ease with which they are recognized, to serve as lines of reference, and distinguished them by the letters B, C, D, E, F, G, H.

Fig. 67′ is a representation of the solar spectrum, in which the most important lines are laid down from accurate measurement.

Later experiments have shown that these lines exist not only within the visible spectrum, but beyond its limits as ordinarily formed. By employing extraordinary precautions, Doctor BREWSTER was able to enumerate two thousand of them.

The results of numerous experiments by FRAUNHOFER, and more recently by KIRCHHOFF, BUNSEN and others, on light from many independent sources, as the sun, some of the most brilliant of the fixed stars, artificial flames, etc. are:

1st. That each species of light produces a spectrum peculiar to itself, in which the number, order, and intensity of the lines, and their positions with respect to the colored spaces, are independent of the substance of the prism, and of its refracting angle.

2d. That any light, having the sun for its ultimate origin, as the light of the clouds, the rainbow, the moon, or the planets, produces a spectrum identical with that formed by direct solar light.

3d. That the light which emanates from incandescent solids or liquids, as the carbon points of

the electric lamp, a platinum wire heated to whiteness, etc. produces continuous spectra; that is, spectra, which, while exhibiting the colors of the solar spectrum, contain no dark lines.

4th. That light from artificial flames, as those of oil, alcohol, hydrogen and its carburets, metallic vapors, etc. produces spectra which are crossed by bright instead of dark lines; that is, by narrow spaces in which the rays appear to be more vivid than those in the adjacent parts of the spectrum, each species of light producing a system of lines peculiar to itself.

5th. That the spectrum produced by electric light is also crossed by bright lines.

6th. That if the light which would form a continuous spectrum if analyzed alone, be made to pass through the flame of a metallic vapor before falling upon the prism, the resulting spectrum will be crossed by dark lines whose indices of refraction are the same as would be those of the bright lines of the spectrum formed by the flame alone. Thus, for example, the yellow light of incandescent sodium vapor gives a spectrum consisting mainly of a double line or band of a brilliant yellow color; and if the light from an incandescent platinum wire be made to pass through this flame, and then be analyzed, the yellow rays of the platinum spectrum, which have the same refrangibility as those of the yellow

bands of the sodium spectrum, will be wanting in the resulting spectrum. Here the yellow rays proceeding from the incandescent platinum are absorbed by the yellow flame of the sodium, and the brilliant lines of the sodium spectrum, though absolutely not less intense than before, become relatively so, and appear dark compared with the other parts of the platinum spectrum. This result may be thus enunciated: an incandescent gas or vapor absorbs rays of the same kind as those which it itself emits.

The principle just stated furnishes an explanation of the existence of dark lines in the solar spectrum. The sun is conceived to consist of a solid or liquid incandescent nucleus, surrounded by a gaseous envelope or photosphere derived from the volatilization of substances contained in the nucleus. The photosphere alone would give a spectrum crossed by bright lines; the nucleus alone, a continuous spectrum, and one of much greater brilliancy than that of the photosphere: as it actually is in nature, the rays from the nucleus which have the same refrangibility as those of the bright lines of the photosphere spectrum are absorbed by the photosphere, and thus is produced a system of lines which seem dark in contrast with the general brilliancy of the spectrum, constituting the dark lines of the solar spectrum.

It has been found that the rays which form the bright lines in the spectra of many metallic flames

have the same refrangibility as those corresponding to some of the dark lines of the solar spectrum. Thus in the spectrum of the incandescent vapor of iron, about sixty bright lines have been discovered, each of which has its corresponding dark line in the solar spectrum. It has hence been inferred that vapor of iron exists in the atmosphere of the sun, and that iron is therefore one of the constituents of that luminary. We have similar evidence that the sun's atmosphere contains also the metals sodium, potassium, magnesium, nickel and chromium.

The invariability of the bright lines in the spectra of incandescent metallic vapors, and their production, though the amount of vapor be almost inappreciable, furnish the chemist with a most delicate method of detecting these substances. The instrument employed in this method of research is called the *spectroscope.*

An important practical consequence of the existence of the dark lines of the spectrum, is, that they afford the means of fixing, with the utmost precision, the indices of refraction of the different classes of rays, by determining, instead of the mean rays of the colored spaces, the positions of which are necessarily to some extent arbitrary, those of the lines B, C, D, etc.

A few of the results of FRAUNHOFER'S experiments are given in the following table.*

INDICES OF REFRACTION FOR THE SEVEN FIXED LINES, B, C, D, ETC. OF THE SOLAR SPECTRUM.

SUBSTANCES.	B	C	D	E	F	G	H
Water,	1.3309	1.3317	1.3336	1.3359	1.3378	1.3413	1.3442
Oil of turp.	1.4705	1.4715	1.4744	1.4784	1.4817	1.4882	1.4938
Cr'n. glass,	1.5258	1.5268	1.5296	1.5330	1.5361	1.5417	1.5466
Flint glass,	1.6277	1.6297	1.6350	1.6420	1.6483	1.6603	1.6711

We shall continue to use the terms *indices of the mean and extreme rays*, it being understood that by them we mean the indices of the rays corresponding to the appropriate fixed lines.

70. *The dispersion of light, and the dispersive power of a medium.*

The separation of a beam of compound light into its elementary rays, considered in the preceding articles, is called the *dispersion of light*. The property by which a medium effects this dispersion is called the *dispersive power* of the medium.

Different media possess different dispersive powers. To illustrate this principle, let ABC, A′B′C′ [Fig. 68] be two prisms, the first of flint, the second of crown glass; LI and L′I′ two indefinitely small beams of white light, incident upon them, at the same angle; and I″V, I″R, I‴V′, I‴R′, the extreme emergent rays. Then,

1st. When the refracting angles of the two prisms are equal, it is observed that both I″R and I″V

*The indices given in the table, in Art. 49, are those of the yellowish green rays, it having been agreed to consider them as the mean rays of the spectrum.

suffer a much greater deviation than I‴R′ and I‴V′; and that of the two angles RI″V, R′I‴V′, which determine the extent of the spectra, the former, that belonging to the prism of flint glass, is much the greater.

2d. When the refracting angle of the prism of crown glass is so much increased as to render the deviation of the red rays in both cases equal, it is still observed that the deviation of I″V exceeds that of I‴V′.

3d. When the refracting angle of the second prism is still farther increased, so as to render the deviations of certain other corresponding rays equal, the mean rays, for example, it is found that, while the deviation of I″V continues greater than that of I‴V′, the deviation of I″R is less than that of I‴R′.

Thus, in each of the above cases, the prism of flint glass produces a spectrum of greater angular extent, and of greater length, than the prism of crown glass: flint glass is, therefore, said to have a greater dispersive power than crown glass.

The dispersive power of a medium is measured by the function

$$\frac{n_{\mathrm{v}} - n_{\mathrm{r}}}{n - 1},$$

in which n_{v}, n_{r}, and n, denote the indices of refraction of the extreme violet, the extreme red, and the mean rays, respectively.*

* See note to this article at the end of the chapter.

The numerator of the above function, viz. $n_v - n_r$, is called the *coefficient of dispersion.*

In general, the most refractive substances are the most dispersive; but there are many exceptions.

71. *The irrationality of dispersion.*

When spectra formed by prisms of different media are compared, it is observed that in general the lengths of the different colored spaces are not proportional to the total lengths of the spectra; thus, when the refracting angles of two prisms of different media are so related as to produce spectra of equal lengths, it is found that the colored spaces in the one are not generally equal to the corresponding colored spaces in the other.

This want of proportionality of the colored spaces in spectra formed by different media, is called the *irrationality of dispersion.* An important consequence of its existence is, that in general the ratio of the dispersive powers of any two substances varies with the rays employed in determining these powers; thus, for example, the red and violet rays will give a different ratio from that obtained by using the orange and indigo.

72. *Achromatism.*

The separation of the rays that accompanies refraction by a prism, causing the object to appear bordered with colored fringes, and hence producing indistinctness of vision, it is natural to enquire whether it is not possible for the deviation to take

place, unaccompanied by the phenomenon of color; in other words, whether the rays of the different colors may not be made to enter the eye parallel to each other, without being rendered parallel to the incident beam.

The English artist, DOLLOND, answered this question in the affirmative, by showing that light emerged almost colorless, though considerably deviated, from a system consisting of two prisms, the one of flint, the other of crown glass, when their refracting angles were in a *certain ratio*, and turned in *opposite directions*.

Subsequently the more general result was obtained, both by direct experiment and by mathematical investigation, that, at small angles of incidence light will emerge colorless, or nearly so, from a system composed of two prisms of different dispersive powers, applied face to face, and having *small* refracting angles turned in *opposite directions*, when *the refracting angles are inversely as the coefficients of dispersion.**

Such a combination of refracting media is said to be *achromatic;* and the principle of the method by which the color is annihilated, is called *achromatism.*

73. *The chromatic aberration of lenses. Achromatic lenses.*

It is evident that the same effect, in relation to color, that attends refraction by a prism, must also accompany refraction by a lens. Accordingly we

* See note to this article at the end of the chapter.

find that when a beam of solar light LII′L′ [Fig. 69] is incident upon a converging lens in a direction, for example, parallel to its axis, the different colored rays are, on account of their different refrangibilities, converged to foci distributed along the axis from V the focus of the violet and most refrangible rays, to R the focus of the red and least refrangible, producing a violet image of the sun at V, a red image at R, and images of the other colors of the spectrum at intermediate points; so that the resulting image, white indeed at the centre, is bordered with colored fringes, and consequently appears confused and indistinct.

This new species of aberration, which depends not upon the form of the lens, but upon the different refrangibilities of the elementary rays, is called the *aberration of refrangibility*, or *chromatic aberration.*

The chromatic aberration of a lens being fatal to distinctness of vision, it becomes an important problem to ascertain whether it can be destroyed; and if its destruction be possible, to investigate a method of effecting it. That a lens may be rendered achromatic, is shown by the fact that achromaticity is possible in the case of prisms; for a lens is only an assemblage of infinitely small prisms, symmetrically arranged about a common axis. We, therefore, proceed to investigate the conditions essential to the achromaticity of a lens.

Let OO′, NN′ [Fig. 70] be two lenses of different dispersive powers, placed in contact, and having a common axis AR.

Let the radii of curvature of the first lens be denoted by r, r'; of the second, by r'', r'''.

Let the indices of refraction of the red, violet and mean rays for the first lens be denoted by n_r, n_v, n; for the second, by n'_r, n'_v, and n'.

Then, the principal focal distances of the two lenses for the red rays being denoted by f_r and f'_r, we have

$$\frac{1}{f_r} = \frac{n_r - 1}{r} + \frac{n_r - 1}{r'},$$

$$\frac{1}{f'_r} = \frac{n'_r - 1}{r''} + \frac{n'_r - 1}{r'''}.$$

Assuming

$$\frac{1}{r} + \frac{1}{r'} = \mathrm{F}, \quad \text{and} \quad \frac{1}{r''} + \frac{1}{r'''} = \mathrm{G},$$

we have

$$\frac{1}{f_r} = (n_r - 1)\,\mathrm{F},$$

$$\frac{1}{f'_r} = (n'_r - 1)\,\mathrm{G}.$$

Let the object be at an infinite distance in front of the lens OO′, and let the principal focus of this lens for red light be at R; then will the red rays leave the second surface of the lens OO′, and fall upon the lens NN′ converging to this point, the distance of which from the lens, viz. AR, we have denoted by f_r.

We have then the case of a converging pencil of rays incident upon the lens NN′. The relation between the conjugate focal distances for these rays, is given by the general equation

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f};$$

and, in the present case, we have

$$p=-f_{\mathrm{r}},\quad f=f'_{\mathrm{r}};$$

and supposing the conjugate focus to be at R′, and neglecting the thickness of the lens,

$$q=\mathrm{AR}'.$$

Hence, by substitution, we get

$$-\frac{1}{f_{\mathrm{r}}}+\frac{1}{\mathrm{AR}'}=\frac{1}{f'_{\mathrm{r}}},$$

or

$$\frac{1}{\mathrm{AR}'}=\frac{1}{f_{\mathrm{r}}}+\frac{1}{f'_{\mathrm{r}}}$$

$$=(n_{\mathrm{r}}-1)\,\mathrm{F}+(n'_{\mathrm{r}}-1)\,\mathrm{G};$$

and hence

$$\mathrm{AR}'=\frac{1}{(n_{\mathrm{r}}-1)\,\mathrm{F}+(n'_{\mathrm{r}}-1)\,\mathrm{G}}.$$

We have thus determined the distance at which the red image is formed by the double lens.

If we suppose the focus of the double lens for the violet rays to be at V′, the distance at which the violet image is formed is evidently given by the equation

$$\mathrm{AV}'=\frac{1}{(n_{\mathrm{v}}-1)\,\mathrm{F}+(n'_{\mathrm{v}}-1)\mathrm{G}}.$$

But if the combination is to be achromatic, at least for the red and violet rays, the red and violet images must coincide; that is, we must have

$$AR' = AV'.$$

This condition gives us

$$(n_r - 1)F + (n'_r - 1)G = (n_v - 1)F + (n'_v - 1)G.$$

Reducing, we find

$$(n_v - n_r)F + (n'_v - n'_r)G = 0;$$

and, substituting the values of F and G,

$$(n_v - n_r) \times \left(\frac{1}{r} + \frac{1}{r'}\right) + (n'_v - n'_r) \times \left(\frac{1}{r''} + \frac{1}{r'''}\right) = 0;$$

or, transposing and dividing,

$$\frac{\frac{1}{r} + \frac{1}{r'}}{\frac{1}{r''} + \frac{1}{r'''}} = -\frac{n'_v - n'_r}{n_v - n_r}.$$

Multiplying each member of this equation by $\frac{n-1}{n'-1}$, we get

$$\frac{n-1}{n'-1} \times \frac{\frac{1}{r} + \frac{1}{r'}}{\frac{1}{r''} + \frac{1}{r'''}} = -\frac{n'_v - n'_r}{n_v - n_r} \times \frac{n-1}{n'-1},$$

or

$$\frac{\frac{n-1}{r} + \frac{n-1}{r'}}{\frac{n'-1}{r''} + \frac{n'-1}{r'''}} = -\frac{\frac{n'_v - n'_r}{n'-1}}{\frac{n_v - n_r}{n-1}}.$$

If we denote the focal distances of the two lenses for mean rays by f and f', we have

$$\frac{1}{f}=\frac{n-1}{r}+\frac{n-1}{r'}, \text{ and } \frac{1}{f'}=\frac{n'-1}{r''}+\frac{n'-1}{r'''};$$

and hence the above equation becomes

$$\frac{f'}{f}=-\frac{\dfrac{n'_v-n'_r}{n'-1}}{\dfrac{n_v-n_r}{n-1}}.$$

The first member of this equation is the ratio of the principal focal distances of the two lenses for the mean rays, and the second member is the ratio of their dispersive powers. Hence, in order that *a double lens may be achromatic so far as the red and violet rays are concerned, the principal focal distances of the two lenses composing it must be directly as their dispersive powers.*

The negative sign with which the second member is affected, shows that *the two lenses must be of opposite characters, viz. the one converging and the other diverging.*

The media usually employed are flint and crown glass; and as the dispersive power of flint glass is the greater, the lens of crown glass will have the shorter focal distance, and be the more powerful.

If the combination is required to be converging, so that a real image may be formed, the more powerful lens, that is, the least dispersive, must also be converging.

A double lens, composed of a converging lens of crown glass and a diverging lens of flint glass, of which the principal focal distances are directly as the dispersive powers, answers the above conditions.

But when the aberrations of the extreme red and violet rays are thus corrected, the colored fringes, though greatly reduced in extent, are still apparent, forming what is termed a *secondary spectrum.*

This spectrum is due to the irrationality of dispersion, which, since it causes the ratio of the dispersive powers to vary with the rays employed to determine these powers, makes it necessary that the ratio of the focal distances of the lenses should vary with the images it may be desired to render coincident.

To destroy, or rather sufficiently to diminish, this secondary spectrum, a third lens may be added to the combination. At present, however, the most skilful opticians use but two lenses; and to obtain the dispersive powers to which their focal distances are to be made proportional, they employ, instead of the extreme red and violet rays, the rays which, like those corresponding to the fixed lines D and F for example, would, by their color and intensity, most sensibly affect the eye.

When the conditions of achromaticity are satisfied, we may consider the quantities f and f' in the equations

$$\frac{1}{f} = \frac{n-1}{r} + \frac{n-1}{r'}, \quad \frac{1}{f'} = \frac{n'-1}{r''} + \frac{n'-1}{r'''},$$

as determined.

We have then two equations, each of two unknown quantities, r, r'; r'' r'''; and from the values of these quantities which satisfy the two equations, we can select such as will effect the destruction of the spherical aberration.

A compound lens may thus be constructed, sensibly free from both species of aberration.

NOTES.

NOTE TO ARTICLE 70.

THE deviation of a ray of light by transmission through a prism, is given [note to Art. 48] by the equation

$$D = \sin^{-1}(n \sin R) + \sin^{-1}(n \sin (G - R)) - G,$$

in which the angles of deviation and refraction, and the refracting angle of the prism, are denoted by D, R and G respectively.

In the case in which the refracting angle is very small, and the light falls almost perpendicularly upon the prism, we have, since R also is small,

$$D = n R + n (G - R) - G,$$
$$= (n - 1) G.$$

That is, when the above conditions are fulfilled, the deviation is proportional to the refracting angle of the prism.

This is also true when the position of the prism is such as to render the deviation a minimum (the usual position in experiments with prisms); since, in that case, when G is small, the angle of incidence is also necessarily small.

In article 70, we have said that the dispersive power of a medium is measured by the function

$$\frac{n_v - n_r}{n - 1}.$$

The propriety of this will now be shown.

Let ABC [Fig. 71] be a prism of *very small refracting angle*, and LI a beam of light incident upon it *almost at right angles*; and suppose the extreme and mean emergent rays to intersect the ray LI produced, sensibly in the same point I′. Then, denoting

the angles of deviation, and the indices of refraction of the mean and extreme rays by D, D_r, D_v, n, n_r, n_v, respectively, we have

$$OI'M = D = (n - 1)G,$$

$$OI'R = D_r = (n_r - 1)G,$$

$$OI'V = D_v = (n_v - 1)G.$$

Subtracting the second equation from the third, dividing by the first, and denoting the ratio by Δ, we have

$$\Delta = \frac{OI'V - OI'R}{OI'M} = \frac{D_v - D_r}{D},$$

$$= \frac{(n_v - 1)G - (n_r - 1)G}{(n - 1)G}$$

$$= \frac{n_v - n_r}{n - 1}.$$

That is, the ratio of the angular magnitude of the spectrum, to the deviation of the mean ray, is constant.

For a prism of a different medium, other circumstances remaining the same, we have

$$\Delta' = \frac{D'_v - D'_r}{D'} = \frac{n'_v - n'_r}{n' - 1}.$$

Now since Δ and Δ' are constant, we may, without affecting their values, suppose the refracting angles of the two prisms to be so related as to produce equal deviations in the mean rays, giving

$$D = D';$$

and hence,

$$\Delta : \Delta' :: D_v - D_r : D'_v - D'_r.$$

But when the deviations of the mean rays are equal, the numerators $D_v - D_r$, $D'_v - D'_r$, or the angular magnitudes of the spectra may be considered as bearing to each other the same ratio as the dispersive powers; hence the dispersive powers are to each other as the ratios Δ and Δ'.

NOTE TO ARTICLE 72.

WHEN a ray of light is transmitted through a series of prisms of different media, and small refracting angles, nearly at right angles to each prism, the total deviation is given by the equation

$$D = (n - 1)A + (n' - 1)A' + (n'' - 1)A'' + \text{etc.}; \ldots [\mathrm{I}]$$

in which A, A', A'', etc. are the refracting angles of the prisms; n, n', n'', etc. are the indices of refraction of any of the elementary rays, the mean rays for instance, and D is the total deviation.

Let ABC, DFE [Fig. 72] be principal sections, by the same plane, of two prisms, placed near each other, of small refracting angles A and A', and constructed of media of which the indices of refraction are n and n'.

Let LI be a ray of light incident almost at right angles, passing through the prisms and the thin intermediate stratum of air, in the directions II', I'I'', I''I''', and emerging in the direction I'''E'; and let the several lines be produced as in the figure.

Then, denoting the deviation GOO'' by D, we have

$$D = OO'O'' + OO''O';$$

but OO'O'' is the partial deviation produced by the first prism, and is given by the equation

$$OO'O'' = (n - 1)A;$$

and OO''O' is the partial deviation produced by the second prism, and is given by the equation

$$OO''O' = (n' - 1)A';$$

hence, $$D = (n - 1)A + (n' - 1)A'.$$

This is evidently true, however thin the intermediate stratum of air.

In like manner it may be shown that the deviation produced by three prisms, the conditions remaining the same, will be given by the equation

$$D = (n - 1)A + (n' - 1)A' + (n'' - 1)A'';$$

and, generally, that for each prism added to the system, a term of the form $(n - 1)$ A must be added to the deviation. Hence, the total deviation for any number of prisms is given by equation [I].

The above result has been obtained on the supposition that the refracting angles of all the prisms have the same general direction. When any of them are turned in an opposite direction, the corresponding deviations become negative.

The reason of this will appear from an inspection of figure 73, in which the two prisms ABC, DEF have their refracting angles in opposite directions, and the total deviation O′OO″ is given by the equation

$$\begin{aligned} \mathrm{O'OO''} &= \mathrm{GO'O''} - \mathrm{O'O''O} \\ &= (n - 1)\,\mathrm{A} - (n' - 1)\,\mathrm{A'}. \end{aligned}$$

Now let the conditions essential to the achromaticity of a double prism be required, the refracting angles of the prisms composing it being small, and the light supposed to fall upon the first prism nearly at right angles.

Employing the usual notation, we have, for the deviations of the extreme rays,

$$\mathrm{D_r} = (n_r - 1)\,\mathrm{A} + (n'_r - 1)\,\mathrm{A'},$$

$$\mathrm{D_v} = (n_v - 1)\,\mathrm{A} + (n'_v - 1)\,\mathrm{A'}.$$

But in order that the light may emerge colorless, the elementary rays must be parallel to each other; and therefore the deviations $\mathrm{D_r}$ and $\mathrm{D_v}$ must be equal.

Hence we have

$$(n_r - 1)\,\mathrm{A} + (n'_r - 1)\,\mathrm{A'} = (n_v - 1)\,\mathrm{A} + (n'_v - 1)\,\mathrm{A'};$$

and by reduction,

$$(n_v - n_r)\,\mathrm{A} = -(n'_v - n'_r)\,\mathrm{A'};$$

and hence

$$\frac{\mathrm{A}}{\mathrm{A'}} = -\frac{n'_v - n'_r}{n_v - n_r}.$$

That is, *the refracting angles must be inversely as the coefficients of dispersion, and turned in opposite directions :* the principle given in article 72, as the result of experiment.

This condition, however, ensures the parallelism of the red and violet rays only; and as, in consequence of the irrationality of dispersion, the ratio $\frac{n'_v - n'_r}{n_v - n_r}$ in general changes its value, if we substitute the indices of rays of other colors for those of the red and violet, it is evident that the combination in question will not be entirely free from color ; that there will remain, as in the case of lenses [Art. 72], a secondary spectrum.

But though a double prism cannot be rendered perfectly achromatic, a near approximation to achromaticity may be attained, by selecting, as the rays to be rendered parallel, those which possess high illuminating power, and, at the same time, differ much in color. The rays corresponding to the fixed lines D and F, answer these conditions.

By combining three prisms, the approximation may be carried still farther. The equations for this case, using the former notations, are

$$D = (n - 1)\,A + (n' - 1)\,A' + (n'' - 1)\,A'',$$

$$D_r = (n_r - 1)\,A + (n'_r - 1)\,A' + (n''_r - 1)\,A'',$$

$$D_v = (n_v - 1)\,A + (n'_v - 1)\,A' + (n''_v - 1)\,A'';$$

and by making

$$D = D_r, \quad \text{and } D = D_v,$$

we shall cause the extreme and mean rays to emerge parallel.

We shall thus have two equations between the three quantities A, A′ and A″; any one of which being assumed at pleasure, the other two may be determined.

Here, as in the preceding case, the degree of achromaticity attained, will depend very much upon the rays which are rendered parallel. Those corresponding to the fixed lines C, E and G, answer the required conditions.

CHAPTER IV.

OF THE EYE, AND OPTICAL INSTRUMENTS.

74. *The Eye.*

The exterior envelope of the human eye, a horizontal section of which is exhibited in figure 74, consists of two membranous substances, called the *sclerotica* and the *cornea*. The former $dd'd''$, to which are attached the muscles whereby the eye-ball is moved, is opaque, and of a spherical form. The latter ded'', forming the anterior portion of the envelope, is transparent, and its form that of a segment of a prolate spheroid. The sclerotica is lined with a second membrane, called the *choroid coat*, the inner surface of which is covered with a black velvety substance called the *pigmentum nigrum*. On this lies the *retina*, the third and innermost coat, a very delicate reticulated membrane, consisting of exceedingly minute fibres branching from the optic nerve. This nerve, proceeding directly from the brain, enters the eye obliquely at a point O on the side next the nose.

At the points d, d'', near where the sclerotica and cornea are united, the choroid is divided into two folds ss' $s''s'''$, to the anterior of which is attached

an opaque circular plane gg' called the *iris*. The color of the iris varies through different shades of gray, blue and brown, and determines the color of the eye. In the centre of the iris is a small circular aperture, called the *pupil*, by which light is admitted to the interior of the eye. The pupil is capable of being enlarged or diminished by the action of the muscular fibres of which the iris is composed. This action is involuntary, and is determined by the intensity of the light which falls upon the eye.

A little behind the iris is suspended a transparent, jelly-like substance LL′, called the *crystalline lens*, of the form of a double convex lens of unequal radii, the radius of the anterior surface being the greater. It is contained in a transparent membranous capsule or bag, which is attached to the posterior fold $s''s'''$ of the choroid, and divides the eye into two very unequal parts called the *anterior* and *posterior chambers*. The anterior chamber A is filled with a transparent liquid called the *aqueous humour;* the posterior B, with another transparent liquid, a little more viscid than the former, called the *vitreous humour*. These liquids are contained, like the crystalline, in very thin transparent capsules. That which contains the vitreous humor is called the *hyaloid*. In density and refractive power, the aqueous and vitreous humors differ very little from water: indeed they consist principally of pure water, containing in solution very small quantities

of albumen, gelatine and muriate of soda. The density of the crystalline lens is rather greater than that of water, and increases from the edge to the centre, a formation which obviously tends to correct the spherical aberration of the eye. It also contains a much larger proportion of albumen and gelatine than the aqueous and vitreous humors.

Having thus generally explained the structure of the eye, we will now consider the modifications which light undergoes in traversing the several media of which this organ is composed.

75. Of a pencil of light proceeding from any point of a luminous object, to the eye, the outer rays fall upon the sclerotica, and are reflected; while the central rays pass through the cornea and enter the aqueous humor. Of the latter, the more divergent are intercepted by the iris, which they serve to illuminate; the remaining rays are admitted by the pupil to the interior of the eye, are transmitted through the crystalline lens and the vitreous humor, and finally fall upon the retina. The several media through which the light has passed, acting as a converging system of lenses, it will readily be perceived that they will, under certain circumstances, cause the rays to converge to a focus at or very near the retina, and form there an image of the luminous point. The same being true of all the diverging pencils proceeding from the object, it is evident that an image of the whole

object will be formed at the back part of the eye; and that as the axes of the pencils cross each other before reaching the retina, the image will be inverted with respect to the object. This conclusion may readily be verified by paring away a portion of the posterior part of the eye of a recently killed animal till it becomes translucent: small inverted pictures of objects, viewed through the eye, will then be seen depicted on the back part of it, as upon a screen of ground glass.

It is found to be generally true, that in every case of distinct vision a small inverted image of the object is thus formed at or near the retina, an impression of which is conveyed to the brain by means of the optic nerve. This is the whole of our knowledge of the material part of the process of vision.

The relation between the physical impression of the object and the consequent affection of the mind by which we are enabled to see it, is entirely unknown, and indeed lies beyond the scope of our inquiries.

76. *Aberrations.*

In the curvatures of the cornea and crystalline and in the iris, operating as a diaphragm with a variable aperture, the eye appears to be furnished with the means of correcting spherical aberration. That it is not rigorously achromatic, we learn from actual experiment. The dispersion, however, is insensible except when special means are taken to render it apparent.

77. *Accommodation of the eye to different distances.*

The eye possesses the property of accommodating itself to objects at very different distances: thus we can see objects with sufficient distinctness at the distance of a few inches, and of many miles. Various explanations of the manner in which this accommodation is effected, have been given. The most satisfactory is that which refers it principally to a modification of the crystalline lens. The structure of the crystalline is found to be fibrous. It may hence be regarded as a muscle, having the power of changing its form and density by the action of its fibres, and consequently its focal distance as a lens. The correctness of this view has been confirmed by recent experiments, in which the convexity of the crystalline was found to increase when the eye was suddenly withdrawn from a distant object and directed to one near at hand.

78. *Distance of distinct vision.*

Extensive as is the range of vision, still there is for all eyes a certain distance at which a given object, a page printed in characters of the usual size, for example, is seen with most distinctness. It is called the *distance of distinct vision,* and, for ordinary eyes, varies from eight to twelve inches.

79. *Optic axis, and optic centre of the eye.*

The axis of the eye, or the *optic axis,* is the straight line drawn through the centre of the pupil perpendicular to its plane. When the eye is intently directed towards a luminous point, this axis coin-

cides with the axis of the pencil of diverging rays proceeding from the point to the eye. The *optic centre* of the eye lies on the optic axis, a little behind the crystalline lens.

80. *Visual angle, and apparent magnitude.*

The straight lines drawn from the extremities of an object to the optic centre of the eye, determine an angle called the *visual angle.* If these lines be produced to meet the retina, an equal angle is formed at the vertex, which, as it is subtended by the image on the retina, is called the *apparent magnitude* of the object. It evidently varies both with the real magnitude of the object, and its distance from the eye.

81. *Optic angle.*

When both eyes are fixed upon the same point, the angle which the optic axes make with each other by their intersection at the point is called the *optic angle.* It evidently diminishes as the distance of the point increases, and, when the distance is great, may be regarded as zero.

82. *Estimate of distance.*

Within certain limits, the human eye is capable of estimating distance with considerable accuracy. This estimate appears to be chiefly the result of experience, in connexion with a consciousness that a change in the optic angle, and certain adjustments of the eye, take place in the act of viewing objects which are at unequal distances from us. The judg-

ment is aided by the greater or less distinctness with which the object is seen, and also by our knowledge of its real magnitude when that is known. When the object is distant, and the optic angle is consequently very small, the estimate, depending much upon our appreciation of this angle, is unavoidably erroneous.

83. *Estimate of real magnitude.*

Our estimate of the real magnitude of an object is in general a consequence of its apparent size, and the judgment we form respecting its distance. It is obviously subject to many illusions, and in extreme cases is entirely inaccurate.

84. *Single vision from two images.*

In ordinary vision with both eyes, two images are formed on *corresponding parts* of the retina; but the object nevertheless appears single, because we have learned from experience to infer the existence of but one object from two images thus placed. If, by any means, as for instance the application of pressure, we derange the optic axis of one of the eyes, the two images are no longer formed on corresponding parts of the retina, and the object appears double.

85. *Erect vision from an inverted image.*

The question how erect vision results from an inverted image, has been much discussed. We will only remark that the formation of the inverted image is one step in the complex process by which we are made aware of the exterior existence of the

object, and that it is doubtless in the same process that we learn also its true position.

86. *Persistence of impressions on the retina.*

The impression made upon the retina continues an appreciable time after the cause producing it has ceased to act. This may be inferred from the well known experiment, in which luminous rings are produced by giving a rapid motion to an ignited coal. The impression is not made instantaneously, but a sensible time is required for its production. The mean duration of the impression is about one-eighth of a second. The time during which it remains at its maximum is exceedingly short. The total duration of the impression increases with the intensity of the light.

The fact of the persistence of impressions on the retina furnishes an explanation of various optical illusions: it is also the fundamental principle of an instrument devised for measuring the velocity of the electric fluid.

87. *Accidental colors.*

If, after a strong impression has been made upon the retina by intently viewing a colored object, the eyes are either directed towards a white surface, or suddenly closed, an image of the object, of a color different from that of the object itself, will be seen for a short time: this color is called the *accidental color* of the color of the object. These two colors are always found to be complementary to each other.

Thus the accidental colors of red, yellow, blue and black, are their complementary colors, green, violet, orange and white, respectively; and conversely, the accidental colors of green, violet, etc. are red, yellow, etc. Sometimes the accidental image repeatedly vanishes and reappears, or it vanishes and the direct image reappears, and the alternation is repeated several times before the final disappearance.

88. *Accidental areolas.*

When objects of different colors are placed side by side, the colors are affected in the same manner in which they would be were each object surrounded by a fringe of the color complementary to its own. Thus, if two narrow strips of paper, the one yellow, the other red, are placed very near each other, the red will appear to incline to violet, the complementary color of yellow, and the yellow to green, the complementary color of red. When for the red and yellow, other colors are substituted, as blue and orange, orange and black, blue and white, or white and black, analogous effects are produced. When the colors are complementary, they render each other more vivid and intense. From these and other analogous experiments, it appears that the contour of a colored body must be regarded as surrounded by a fringe of a color complementary to the color of the body, which, from its faintness, is not perceived except in circumstances peculiarly favorable to its visibility. This fringe is called an

accidental areola. In some instances, a very faint areola of the same color as the object has been seen a little beyond the first.

It can readily be conceived that the subject of accidental areolas is susceptible of useful applications in those arts in which it is desired to modify or heighten the effect of colors, and indeed in all cases in which their mutual influence is to be regarded, as in the selection of furniture or dress, or in the arrangement of flowers in a parterre or bouquet.

88′. *Irradiation.*

A white or vividly colored object always appears larger than a black or sombre object of the same dimensions. Thus, of two equal circles, the one white on a black ground, and the other black on a white ground, the first appears larger than the second. This phenomenon is called *irradiation.*

88″. The cause of the persistence of impressions on the retina, of irradiation, and of accidental colors and areolas, exists without doubt in the eye itself. Several hypotheses have been proposed to account for them; but the only explanation which embraces all the facts, is that of M. PLATEAU. In this it is supposed, first, that the action of a beam of light upon the retina causes this membrane to assume alternately opposite states, both in successive periods of time in the part on which the beam of light has fallen, and simultaneously in contiguously surrounding parts of its surface, both kinds of changes taking

place gradually, or, in other words, according to the law of continuity; and, secondly, that these opposite states produce the respective sensations of the real and accidental colors.

Considering first the opposite states that occur in succession, which are supposed to follow the sudden withdrawal of a luminous object upon which the eye has been intently fixed, and which we may conceive to be caused by a process analogous to the oscillation of a body about its position of equilibrium, we perceive that the gradual decrease in the excitement of the retina when the light ceases to act upon it, should give rise to a phenomenon like the persistence of the image; and that the opposite states through which the retina subsequently passes, should produce successively the accidental image, the re-appearance of the original color, and so on.

The simultaneous production of opposite states in surrounding parts of the retina, which is supposed to occur during the action of the light upon the eye, and which we may conceive to be analogous to what takes place in a vibrating plate in which the parts separated by nodal lines are oppositely affected, explains the two latter phenomena. For, the state, assumed by the part of the retina upon which the rays immediately fall, extending a little beyond the contour of this part, an increase of the apparent size of the object should result, constituting the phenomenon of irradiation: a little beyond this, the membrane being in a state opposite to that of

the part originally excited, an accidental areola should appear; and still beyond, another state might exist, identical with that of the central part, producing a faint areola of the primitive color.

89. *Sensibility of the retina.*

The sensibility of the retina is not the same throughout. The part most sensible is that near the axis of the eye; and an object can be seen with entire distinctness only when the image is formed upon it.

The extremity of the optic nerve, from which proceed the minute filaments composing the retina, is entirely insensible to the stimulus of light, conveying no impression to the brain, and is hence called the *punctum cœcum.*

90. *Defects of the eye.*

When the curvatures of some parts of the eye are too great, and the organ consequently possesses too much converging power, the image is formed in front of the retina, and the distance of distinct vision is less than the medium. Persons having such eyes are said to be *short-sighted.*

When, from age or some other cause, the eye loses a portion of its converging power, a result principally due to the flattening of the crystalline, the image is formed behind the retina, and the distance of distinct vision is greater than the medium. Persons whose eyes are thus affected, are said to be *long-sighted.* In the case of either long or short-sightedness, the defects are frequently due in a

greater or less degree to a loss of flexibility in the crystalline, in consequence of which it can no longer adapt itself to objects at different distances.

91. *Optical instruments.*

Having, in the preceding chapters, sufficiently developed the laws which regulate the reflection of light by plain and spherical mirrors, and its transmission through lenses, and having now explained to a certain extent the structure of the eye and the process of seeing, we are prepared to consider the instruments by which sight is modified and extended; by which the defects of the organ, arising from age, accident or original malformation, are effectually remedied; nature and art copied with perfect fidelity, and objects of the profoundest interest rendered visible and measurable, which, by their minuteness or distance, are put without the sphere of unaided vision.

92. *Spectacles.*

The ordinary defects of vision [Art. 90] may be remedied by the use of lenses. Thus, if a person either long or short-sighted wishes to read at the distance of distinct vision for eyes in the normal condition, it is only necessary to interpose between the eyes and the book lenses by which the light may be so refracted as to enter the eye as if it came from the book at the distance required by the imperfect organs. The real and apparent distances of the book are evidently the conjugate focal distances of

the lens employed, the relation between which is given generally by the equation

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}:$$

p, in the present case, denoting the ordinary distance of distinct vision; q, which must be considered negative, the distance adapted to the naked eye, and f the principal focal distance of the lens.

Making q negative, we have

$$\frac{1}{p}-\frac{1}{q}=\frac{1}{f}.$$

In the case of a long-sighted person, q is greater than p, and f is positive; for a short-sighted person, q is less than p, and f is negative. Hence, in the former case, converging lenses must be used; in the latter, diverging lenses, their principal focal distances being determined by the above equation.

The glasses of spectacles are lenses constructed according to the above principles; the oval form being used instead of the circular, because the eye requires a greater extent of field laterally than vertically.

92'. *The Stereoscope.*

When a solid or any object in relief is viewed at a short distance with one eye only, it is found to present to the right eye an appearance a little different from that which it presents to the left. The same difference must obviously exist when the object

is seen with both eyes at the same time. This fact has suggested the construction of the stereoscope. In this instrument, two pictures l and l' [Fig. 74′] of the same object, the one such as it would appear to the right eye, the other to the left, are disposed so that each eye can see only the picture designed for it; and then by means of two converging eye-glasses p and p', one for each eye, the light is so refracted as to enter the eye as if it came from a single object L in an intermediate position; or, in other words, the images formed by the lenses of the two pictures l and l' are superimposed at L.

The conditions of vision are evidently the same as if the object itself were placed at L. The result is, as we should expect, that the object is seen at L in strong relief. The eye-glasses are formed by cutting a double convex lens into two equal parts, in a plane passing through its axis.

93. *The simple microscope.*

When we attempt to examine a very minute object with the naked eye at the ordinary distance of distinct vision, the image formed on the retina is too small to be clearly perceived; and if, to increase its size, we bring the object nearer the eye, the rays, becoming too divergent, are no longer brought to a focus on the retina, and the vision is confused and indistinct. But if we interpose between the eye and object a converging lens of short focal length, giving it such a position that the light refracted by

it may diverge from a virtual image at the distance of distinct vision, the object will then appear clear and magnified. Such a lens thus employed is called a *simple microscope*.

Let RS [Fig. 75] be the lens almost in contact with the eye, LL′ the object placed between the lens and its principal focus, and PP′ the virtual image; then denoting the principal focal distance of the lens by f, O′I (which is nearly equal to the distance of distinct vision) by q, and O′U the distance of the object by p, we shall have the relation

$$\frac{1}{p} - \frac{1}{q} = \frac{1}{f};$$

and hence

$$\frac{q}{p} = \frac{q+f}{f}.$$

Now the object is evidently magnified in its linear dimensions, in the ratio of PP′ to LL′. Denoting this ratio, which is called the linear magnifying power of the lens, by x, we have

$$x = \frac{\mathrm{PP'}}{\mathrm{LL'}} = \frac{\mathrm{O'I}}{\mathrm{O'U}} = \frac{q+f}{f};$$

or, since f is always very small compared with q,

$$x = \frac{q}{f}.$$

From this it appears that the object is magnified nearly in the ratio of the distance of distinct vision, to the focal distance of the lens, and hence, the former distance being supposed constant, as it is for

the same individual, that the magnifying power of the lens is *inversely as its focal distance.*

Lenses for single microscopes are constructed not only of glass, but of several of the gems; as the diamond, garnet, ruby and sapphire. Lenses of garnet have been constructed of focal-length as small as the 50th of an inch.

The superiority of these lenses to those made of glass, is due to the circumstance that the gems possess much greater refractive energy than glass, associated with low dispersive power; so that a diamond lens, for example, may be made to produce a much greater refraction than a glass lens of the same curvature, without any increase of spherical or chromatic aberration. But lenses made of the precious stones can never be generally used, not only on account of the cost of the materials, but of the difficulty with which they are wrought. When glass is employed, and the microscope required to be of very high power, small spheres may be substituted for lenses. Spherules of glass have been produced, having a focal length of only the 190th of an inch.

As the simple microscope is an instrument of much utility in the researches of the naturalist and anatomist, great attention has been directed to its improvement. One of its forms is a combination of two lenses, so adjusted as to operate as a single lens, but with less spherical aberration, called a

microscopic doublet. It consists of two plano-convex lenses, separated by a small but variable interval, and having their plane faces directed towards the object, the aperture being determined by a diaphragm placed between them.

An excellent form of the simple microscope is that known as RASPAIL'S.

94. *The camera obscura.*

The camera obscura consists essentially of a dark chamber or box, having inserted in one of its sides a converging lens, by which miniature pictures of external objects are formed on a screen within, suitably disposed to receive them.

One of the forms of this instrument is represented in figure 76. In this the pencils of light, diverging from an object LL′, and falling upon a plane mirror CD, inclined to the horizon at an angle of 45°, are reflected downwards, and brought to their several foci, by the converging lens AB. An image SS′ is thus formed, which may be exhibited on a screen of ground glass; or, if a sketch of the object be desired, may be received on a sheet of white paper, and the lines of it traced thereon by a pencil. The head and hand of the observer are admitted to the interior of the box by two apertures; and a screen of cloth, falling over him, excludes the light. The lens AB should be achromatic. The mirror may be dispensed with, by using, instead of the lens, a triangular prism, having one of its faces plane, and one

or both of the remaining two curved. The plane surface AC [Fig. 77] performs the part of a mirror, and the surfaces AB and BC act as a converging lens. A form of the camera obscura, in which the axis of the lens AB is horizontal, is the instrument used in photography to form the image on the sensitive surface.

95. *The camera lucida.*

This instrument is used by artists for sketching landscapes, and delineating edifices, objects of natural history, etc.

It consists of a quadrilateral prism of glass ABCD [Fig. 78], right-angled at B, and having the angle D equal to 135°. The remaining angles A and C may have any value, consistent with the obvious condition that their sum must be equal to 135°. In the form originally given to the instrument by Dr. Wollaston, the inventor, these angles are equal to each other.

The vertical face BC being directed towards an object LL′, small pencils of light, emanating from it, and having their axes perpendicular to the face, will enter the prism without refraction, and suffering total reflection from the surfaces CD and AD, finally emerge from the horizontal face AB at right angles to it, and near the edge A. That the pencils emerge at right angles, will appear from examining the course of the axis of one of them, and determining its several angles of incidence and reflection. These

emergent pencils will diverge from a virtual image SS′, which may be seen by an eye placed near A.

If a delineation of the object be required, such a position must be given to the eye, that the pupil *pp′* may be divided into nearly equal parts by the vertical plane passing through the edge of the prism; so that with the anterior part the observer shall see the image; and with the other at the distance of distinct vision, the paper on which it is projected, and the point of the pencil used in tracing it. As in general the rays proceeding from the virtual image have not the same degree of divergence as those from the paper, it is necessary either to place a diverging lens E in front of the prism, or a converging lens B above the paper, by suitably adjusting which the divergence may in both cases be rendered equal.

96. *The solar microscope.*

When a small but very luminous object is placed a little without the principal focal distance of a lens of high power (such as we employ in the single microscope to produce a virtual image of a minute object), a real image will be formed, inverted and greatly magnified, which may be received on a screen at the conjugate focal distance. A lens of this kind, connected with a mirror and one or more lenses for illuminating the object by solar light, forms a solar microscope.

The essential parts of this instrument may be seen in figure 79, in which RR′ represents a plane

mirror, MM′ and NN′ two lenses for illuminating the object, and OO′ the object lens by which it is magnified. A beam of solar light is reflected by the plane mirror in the direction of the common axis of the first two lenses, and concentrated upon the object LL′, which is thus rendered intensely luminous. Then such a position being given to the lens OO′, that its distance from the object may be a little greater than its principal focal distance, a magnified and inverted image SS′ will be formed, and may be exhibited on a screen. A distinct provision exists for illuminating the object when opaque. The electric lamp, a form of the solar microscope in which the light of the voltaic arc is substituted for the solar beam, is now frequently employed in experiments requiring an intense light.

97. *The magic lantern.*

The magic lantern is similar in principle and design to the solar microscope; the light of a lamp being used instead of solar light; and pictures on glass, of relatively large size, being substituted for the minute objects of the microscope.

98. *The compound microscope.*

The design of this instrument is, like that of the simple microscope, to magnify very small objects, and render their form and structure distinctly visible. Its essential parts are two converging lenses, having a common axis; the one called the *object glass;* the other, the *eye-glass.* By means of the

former MM′ [Fig. 80], we produce a magnified and inverted image PP′ of a small but highly illuminated object LL′; and employing the latter as a single microscope, we magnify this image, forming a second and virtual image QQ′ at a distance from the lens nearly equal to that of distinct vision.

It will readily be perceived that the position of the object LL′ must be a little without the principal focal distance of the lens MM′; and that of the image PP′, a little within the principal focal distance of the lens NN′. The linear magnifying power x may be determined by comparing the similar triangles OEL, OIP; O′IP, O′GQ. From the former we get

$$\mathrm{PI} = \frac{\mathrm{IO} \times \mathrm{LE}}{\mathrm{EO}};$$

and from the latter,

$$\mathrm{PI} = \frac{\mathrm{IO}' \times \mathrm{QG}}{\mathrm{GO}'}.$$

Hence we have

$$\frac{\mathrm{IO} \times \mathrm{LE}}{\mathrm{EO}} = \frac{\mathrm{IO}' \times \mathrm{QG}}{\mathrm{GO}'},$$

and hence

$$x = \frac{\mathrm{QG}}{\mathrm{LE}} = \frac{\mathrm{IO} \times \mathrm{GO}'}{\mathrm{EO} \times \mathrm{IO}'}.$$

That is, the distance of distinct vision and the distance IO being considered constant, the magnifying power is very nearly *inversely as the product of the focal distances of the two glasses.*

The circular space visible through the microscope is called the *field of view:* it is generally considered as limited by the rays which pass through the optic centre of the object-glass at such an angle as just to graze the edge of the eye-glass. The two glasses are set in distinct tubes, so constructed that the one may glide within the other. Attached to the frame of the microscope are a mirror and lens for illuminating the object, and also provisions for supporting it, and placing it at the proper distance from the object-glass.

The compound microscope, constructed as we have just described it, without provisions for enlarging the field of view, or correcting either spherical or chromatic aberration, is very defective. By employing another lens and one or more diaphragms, a greater extent of field may be obtained, and the aberrations somewhat diminished. Instead, however, of attempting to indicate any of the intermediate states through which this instrument has passed to its present greatly improved condition, it will be more consistent with our limited plan to give a brief description of it as now constructed by the best opticians. For this purpose, we select the *universal microscope* of M. Chevalier, a distinguished French optician.

99. The optical parts of this instrument may be seen in figure 81, where LL′ represents the object, MN an achromatic object-glass, VAV′ a prism

having its face VV′ inclined at an angle of 45° to the axis OW, and RS and R′S′ two lenses forming a compound eye-glass. The pencils of light from LL′ refracted by the lens MN, and converging to their several foci above the prism, suffer total reflection at the surface VV′; and proceeding in the direction of the axis OW, are received by the lens R′S′, and form the image PP′; this is again magnified by the lens RS, producing the final image QQ′ at the distance of distinct vision.

There are three achromatic object-lenses, of principal focal distances varying from about three-tenths to four-tenths of an inch (8 to 10 millimetres), which may be used singly, or, when higher degrees of power are required, in combination. There are also six eye-glasses: the two of greatest power are single; the others compound, consisting, as represented in the figure, of two plano-convex lenses, their plane surfaces directed towards the eye, and having between them, at the place where the image PP′ is formed, a diaphragm of determinate aperture. When those combinations of the eye and object-glasses are used, of which the linear magnifying power does not exceed 500, the object appears perfectly clear and well defined. Those combinations which magnify from one to four thousand times, produce images slightly confused.

100. *Telescopes.*

If the light proceeding from a distant object be received on a converging lens MM′ [Fig. 82], or a

concave mirror MM′ [Fig. 89], a small and brilliant image PP′ of the object will be formed near F the principal focus of the lens, or F′ that of the mirror. This image will obviously be far more luminous than that of the same object formed on the retina by direct vision, and hence will admit of being more or less magnified. To magnify it, we employ, as in the compound microscope, an eye-glass operating as a simple microscope: an image is thus formed at the proper distance from the eye, and sufficiently luminous for distinct vision. Such a combination of an object-glass and eye-glass forms a *refracting telescope;* of an object-mirror and eye-glass, a *reflecting telescope.*

REFRACTING TELESCOPES.

101. *The astronomical telescope.*

This kind of refracting telescope is, as the name indicates, that which is used by astronomers in observing the heavenly bodies. In its simplest form, it is shown in figure 82, in which MM′ represents the object-glass, a lens of large aperture and considerable focal length; NN′ the eye-glass, a lens of small aperture and short focal distance (the two lenses having a common axis OQ); PP′ the real image, inverted, formed by the lens MM′ very near its principal focus F; and SS′ the virtual image seen through the eye-glass, at the distance of distinct vision. The image PP′ is a little within the

focal distance of the eye-glass; but its distance from the focus of each lens is so small, that we may consider both foci as sensibly coinciding, and the image as formed at the common point. The magnifying power x is the ratio of the angle LO″L′, or its equal NO″N′, under which the object would be seen with the naked eye, to the angle NON′ under which the virtual image is seen through the eye-glass. To determine x, since the pencil PN is sensibly parallel to the axis, we have the angle NOO′ equal to PO′F, and hence

$$x = \frac{\text{PO}'\text{F}}{\text{PO}''\text{F}} = \frac{\text{tang PO}'\text{F}}{\text{tang PO}''\text{F}}$$

$$= \frac{\text{PF}}{\text{O}'\text{F}} \times \frac{\text{O}''\text{F}}{\text{PF}} = \frac{\text{O}''\text{F}}{\text{O}'\text{F}}.$$

That is, the magnifying power is *directly as the focal distance of the object-glass, and inversely as that of the eye-glass.*

A practical limit to the indefinite increase of the magnifying power of the telescope consists in the difficulty of constructing large object-glasses, and the necessity of keeping the virtual image sufficiently brilliant to produce the requisite impression upon the retina. In the above description we have supposed both glasses of the telescope to be single; but as actually constructed, the object-glass is a compound lens, achromatic, and free from spherical aberration: the eye-glass is also compound, consisting of two distinct lenses, of such forms and

powers, and so disposed, that the final image may appear free from color, and well defined.

EYE-GLASSES.

102. *The eye-glass of Campani or Huygens.*

When the telescope is designed merely for viewing objects without reference to measurement, the lenses composing the eye-glass are disposed as in figure 83, where the rays, converging to the image LL′ at the focus of the object-glass, are intercepted by the *field-lens* RR′, and so refracted as to form an image L″L‴ a little within the focal distance of the *eye-lens* NN′, by which the final image is formed. The two lenses are of the same kind of glass, plano-convex, with the plane faces turned towards the eye. Their relative powers and positions are determined by calculation.

103. *The eye-glass of Ramsden.*

When the telescope is to be connected with graduated arcs for measuring angular magnitudes, it is necessary to place in the focus of the object-glass, at right angles to the axis, a system of wires (vertical and horizontal); by means of which, the exact direction of the rays coming from the heavenly body to the eye, at any instant, can be determined.

In the arrangement of Campani, this system would be situated between the field and eye-lens, and any motion of the eye-glass would give rise to certain practical inconveniences. To obviate this difficulty,

an eye-glass is used, which occupies a position in the telescope entirely beyond the focus of the object-glass. It is represented in figure 84.

It is proper to remark, that in the eye-glasses just described, the achromatism is not produced, as in the case of an object-glass, by effecting the coincidence of the images of different colors, but by causing them to be formed of magnitudes proportional to their distances from the eye. Figures 85 and 86 will illustrate this. In figure 85, the images are represented as they are formed by the first lens, with their extremities not in the same straight line; in which case the resulting image, as seen by the eye at O, is bordered with colored fringes. In figure 86, they are represented as corrected by the second lens, having their extremities in the same straight line, and sending to the eye at O′ rays of all the prismatic colors, that is, all the elements of white light at the same time.

104. *The terrestrial telescope.*

The apparent inversion of the object by the astronomical telescope, of little consequence when the heavenly bodies are concerned, is a serious defect when the objects are terrestrial. To remedy this, an eye-glass is employed, consisting of three lenses, as shown in figure 87, in which the points F, O′ and K are the common foci of the lenses RR′, PP′; PP′, NN′; NN′, MM′, respectively.

In this form of the instrument, the small pencils SN, S′N′, diverging from the extremities of the first

real image SS′, are refracted by the lens NN′ into the two beams NP′, N′P; these crossing each other at the common focus O′, and falling upon the lens PP′, are converged to the points Q, Q′, forming at the focus of PP′ the extremities of a second real image, inverted with respect to the first: this second image is magnified by the lens RR′, and an erect virtual image is thus formed at the distance of distinct vision. If, as is usually the case, the lenses NN′ and PP′ have the same focal distance, that is, if $CK = C'F$, we shall have $FQ = KS$; and the magnifying power, or the ratio of the angle at O to the angle at K′, will evidently be, as in the astronomical telescope, equal to $\frac{KK'}{OO''}$.

105. *The Galilean telescope.*

If, in front of the image PP′ [Fig. 88] of a distant object, formed by a converging lens MM′, we place a double concave lens NN′ at a distance from the image a little greater than its principal focal length, the pencils converging to PP′ will be refracted by NN′, so as to diverge from a virtual image RR′ erect and magnified. This form of the telescope was invented by GALILEO. To determine its magnifying power x, we have

$$x = \frac{POQ}{PO'Q} = \frac{\text{tang } POQ}{\text{tang } PO'Q}$$

$$= \frac{PQ}{QO} \times \frac{QO'}{PQ} = \frac{QO'}{QO}.$$

But QO' and QO are the focal lengths of the lenses: hence the magnifying power is the same as in the astronomical and terrestrial telescopes.

This combination of lenses furnishes an example of the case in which rays converging to a point Q behind a diverging lens NN', are so refracted by the lens as to diverge from a point E in front of it, at the distance of distinct vision.

By adapting the general formula

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

to these circumstances, it may be shown that OQ must be greater than the principal focal distance of the lens, and that the distance OO' between the lenses must increase with the distance of distinct vision.

In viewing objects through this telescope, the most advantageous position for the eye is evidently as near as possible to the point F where the axes of the diverging pencils cross each other: it must therefore be placed close to the surface of the eye-glass. The actual field of view is thus limited by the aperture of the pupil.

REFLECTING TELESCOPES.

106. There are several forms of the reflecting telescope. We shall very briefly indicate the construction of each.

107. *The Newtonian reflecting telescope.*

In this form of the instrument, a plane mirror BB′ [Fig. 89], inclined at an angle of 45° to the axis, is placed between the object mirror MM′ and its principal focus F′; and the rays from a distant object, which, after reflection from MM′, would, if not intercepted, form at the principal focus an image PP′, are thus deflected towards the side of the telescope, and the image actually formed is P″P‴ equal to PP′, and parallel to the axis AC. This image is viewed through an eye-glass NN′ placed in an aperture in the side of the instrument, at a distance from the image a little less than its own focal length.

To determine the magnifying power, since the object is seen with the naked eye under the angle LCL′, and through the telescope under the angle NON′, we have

$$x = \frac{\text{NON}'}{\text{PCP}'} = \frac{\text{F}'\text{C}}{\text{OC}'};$$

that is, *equal to the focal distance of the object-mirror, divided by the focal distance of the eye-glass.*

108. *The Gregorian reflecting telescope.*

In the Gregorian telescope, the object-mirror MM′ [Fig. 90], has at its vertex a circular aperture, in which is inserted a tube containing the eye-glass NN′. Opposite this is placed a small mirror BB′, concave, with its concavity directed towards NN′,

and at a distance from MM′ a little greater than the sum of the focal lengths of both mirrors.

The image of a distant object being formed at F′ the principal focus of MM′, and a little without the focal distance of BB′, the rays diverging from it form by reflection from BB′ a second image PP′, inverted with respect to the first, and hence upright. By a proper adjustment of the mirrors, this image is formed near the vertex of the larger one, where it is magnified by the eye-glass.

109. *The Cassegrainian telescope.*

This differs from the Gregorian telescope, in the form and position of the smaller mirror; it being convex, and placed between the object mirror and its principal focus, and at a distance from the latter a little less than its own focal length.

110. *Sir William Herschel's telescope.*

In this telescope, the image formed by the object-mirror is viewed directly by the eye-glass, without the interposition of the smaller mirror. That the head of the observer may intercept as little light as possible, the axis of the telescope is slightly inclined to the direction of the light, so that the image may be formed near the edge of the tube.

Recently M. Foucault seems to have made a great advance in the construction of reflecting telescopes, by substituting for metallic object-mirrors, glass mirrors wrought into the parabolic form, and silvered on the concave surface by a chemical pro-

cess. These are mounted as in the Newtonian reflector, having the eye-glass at an opening in the side of the tube; but in place of the ordinary eye-glass, a system of lenses is employed similar to that in the compound microscope.

110′. *The Fresnel lens.*

Lenses of great size, for the purpose of illumination, are constructed in segments. The central segment is a plano-convex lens, and around it is a system of concentric rings, plane on one side and convex on the side opposite. The lens and rings have their convex surfaces so curved as to refract parallel rays to a common focus, and are cemented by the plane faces to a plane glass. Each ring is generally composed of several segments. A lens thus constructed may have a large aperture, and yet be of moderate thickness, besides being little affected by spherical aberration. This lens was first used by FRESNEL, and is hence called the *Fresnel lens.* A combination of Fresnel lenses, or a modification of them, forms the Fresnel light, which is now rapidly replacing the old apparatus of parabolic mirrors in the illumination of light-houses.

CHAPTER V.

OF LUMINOUS METEORS.

In a systematic arrangement, the subject of luminous meteors would be assigned to meteorology; but the illustrations it affords of the preceding laws are so beautiful and of such general interest, and the examples it presents of the application of mathematical principles to the explanation of natural phenomena, are so striking, that no apology can be required for introducing it here.

111. *The rainbow.*

This well known meteor usually presents the appearance of a single circular arch, composed of concentric bands of the prismatic colors, arranged in the same order as in the solar spectrum. The conditions of its formation and visibility are, 1st, that the sun shall shine during the fall of rain; and 2d, that the observer shall be placed between the drops of rain and the sun, with his back to the latter. The centre of the arch is always in the prolongation of the line drawn from the centre of the sun to the eye of the observer, and the radius of its outer circumference subtends an angle of about 42°. This arch is called the *primary bow*. A second arch, less vivid than the first, and exterior to it, is

sometimes seen. It has the same centre as the first, and the radius of its outer circumference is about 54°. This is called the *secondary bow.* In the primary bow, the violet band is nearest the centre; in the secondary bow, the red band.

From the circumstances attending the appearance of the rainbow, it may be inferred generally that its formation is due to the action of the drops of rain upon the solar rays. To show the exact manner in which it takes place, we must first determine the course of a ray of light in its transmission through a drop of water. Since all the particles of a drop of rain may be considered as equally affected by gravity, its form is determined by molecular attraction alone, and is therefore spherical. Let II′I″I‴ [Fig. 91], be the section of a drop by a plane passing through its centre, and the line SG drawn through the centre of the sun and the eye of the observer; and let S′I be a beam of solar light incident upon the drop at I, in the plane of the section. On account of the distance of the sun, this beam may be considered sensibly parallel to SG; and for the sake of simplicity, we shall suppose it to be homogeneous. A part of it will enter the drop, and undergoing refraction at I, will take a certain direction II′. At I′, another portion will be refracted, and emerge into the air; while the remaining part of the beam will be reflected within the drop to some point I″, where another refraction and reflec-

tion will take place. These refractions and reflections will thus be repeated indefinitely. It hence appears that a ray of light S′I, proceeding from the sun, may have its direction so changed by one or more reflections within the drop, as to reach the eye of an observer at E, whose back is turned towards the sun.

If we suppose the light to be compound, each refraction will evidently be accompanied by dispersion and the production of color. But what determines the convergence of beams of the different colors to the same point E? The answer to this question involves a full explanation of the rainbow.

112. Any ray S′I [Fig. 92], incident upon the surface of a drop, makes with the corresponding emergent ray PI″E a determinate angle of deviation S′PE, which varies with the angle of incidence. Now as the parallel rays of an incident beam fall upon the drop at different angles, they will not in general emerge parallel after the same number of reflections, and will therefore not enter the eye of a distant spectator in sufficient numbers to produce vision.

To determine whether any rays of the same beam, incident upon the anterior surface of the drop, will emerge parallel, we must get an expression for the deviation in terms of the angle of incidence, and calculate its value for a great number of different values of that angle. It will thus

be found that for one internal reflection, the deviation (which is nothing for the ray which falls upon the drop at right angles, and passes through the centre) increases with the angle of incidence, till the latter attains a certain value, depending upon the color of the rays; after which, it diminishes with a further increase of that angle. It will also be found that the rays immediately preceding and succeeding the ray of which the deviation is a maximum, suffer very nearly the same deviation as that ray, and consequently emerge sensibly parallel to it and to each other. The beam of light thus formed, suffering no diminution of its intensity by separation of the rays, is capable of producing the sensation of vision at a great distance. Upon these rays, then, must depend the phenomenon of the rainbow.

The constancy of the deviation of the rays which fall upon the drop very nearly at the angle of incidence corresponding to the maximum deviation, is a general property of the maximum and minimum values of functions. In the case of two internal reflections, the angle of deviation of the rays which emerge parallel to each other is a minimum.

To obtain the maximum and minimum values of the deviations, and the corresponding angles of incidence and refraction, instead of the tedious process of calculating the deviations for a series of incidences from 0 to 90°, as we have supposed

above, we may have recourse to the calculus, and determine the required quantities by a single operation.* The results are given below; I, R and D denoting the angles of incidence, refraction and deviation respectively, and n the index of refraction. In the case of one internal reflection, we have,

For the red rays,

$$\begin{aligned} n &= 1.3333, \\ I &= 59^\circ\ 23'\ 30'', \\ R &= 40^\circ\ 12'\ 10'', \\ D &= 42^\circ\ \ 1'\ 40''; \end{aligned}$$

For the violet rays,

$$\begin{aligned} n &= 1.3456, \\ I &= 58^\circ\ 49'\ 30'', \\ R &= 39^\circ\ 24'\ 20'', \\ D &= 40^\circ\ 16'\ 20''. \end{aligned}$$

In the case of two internal reflections, we have,

For the red rays,

$$\begin{aligned} n &= 1.3333, \\ I &= 71^\circ\ 49'\ 55'', \\ R &= 45^\circ\ 26'\ 50'', \\ D &= 50^\circ\ 58'\ 50''; \end{aligned}$$

For the violet rays,

$$\begin{aligned} n &= 1.3456, \\ I &= 71^\circ\ 26'\ 10'', \\ R &= 44^\circ\ 47'\ \ 7'', \\ D &= 54^\circ\ \ 9'\ 38''. \end{aligned}$$

113. Now through the straight line SG [Fig. 92], supposed to be drawn through the centre of the sun

* See note to this article at the end of the chapter.

and the eye of the observer, conceive a vertical plane to pass. This plane may evidently be supposed to pass also through the centres of a series of drops, *a*, *b*, *c*, *d*, intersecting them in the circles II′I″, I‴I$^{\text{IV}}$I$^{\text{V}}$, etc. Let S′I be a small beam of solar light incident upon the drop *a* in the circle II′I″, at an angle of 58° 49′ 30″. A number of the violet rays of this beam, after one internal reflection, will emerge at I″ parallel to each other, and making with SG an angle GEI″ of 40° 16′ 20″, equal to the alternate angle S′PE, the angle of maximum deviation for violet light. This violet-colored beam will enter the eye at E with its intensity but little diminished by distance. The pupil of the eye being of sensible dimensions, other similar beams, produced by the refraction of the drops (not represented in the figure) immediately above and below *a*, will also enter it; and as each beam will cause the sensation proper to a violet-colored point, the observer will perceive at I″ a vertical line of a violet color, and equal in length to the diameter of the pupil. The remaining rays which emerge at I″, being less refrangible than the violet rays, will, as represented in the figure, fall below the eye.

At some distance above the drop *a*, there will evidently be another drop *b*, in such a position that, a solar beam S″I‴ falling upon it at an angle of incidence of 59° 23′ 30″, the red rays of this beam which will emerge at I$^{\text{V}}$ parallel to each other, and

making with SG an angle of 42° 1′ 40″, will intersect SG at E the place of the eye, so that the observer will see at I^v a small vertical line of a red color. Here the other rays refracted at I^v, will emerge above the eye.

In precisely the same manner, the solar rays incident upon drops between *a* and *b*, in the common direction S′I, will be decomposed; and beams of all the intermediate colors of the spectrum will, after one interior reflection, be so refracted as to enter the eye at E. Thus in the vertical plane passing through SG, there will be visible to the observer at E, a linear solar spectrum, extending from *a* to *b*, and having the violet space nearest SG. But this appearance will not be confined to the vertical plane: a similar spectrum, at the same angular distance from SG, will be seen by the eye at E, in every plane passing through SG and intersecting the drops. The observer at E will therefore see, at the same time, a vast number of spectra, so arranged as to form a circular arch, resembling, in the variety and disposition of the colors, the primary arch of the rainbow.

In like manner it may be shown that light which has undergone two internal reflections within certain drops more elevated than *b*, will emerge in beams of the prismatic colors directed to E, so as to present to the observer at that point another circular arch similar to the secondary bow. An inspec-

tion of the figure will render the circumstances of its formation apparent.

It can be shown that light, which has suffered three internal reflections, may reach the eye of the observer; but on account of the additional reflection, we should expect the bow thus formed to be seldom visible. In confirmation of this, a tertiary bow, though said to be sometimes seen, is a very rare occurrence.

114. The angular breadth of the primary bow is equal to the difference of the angles P′EG, PEG, under which its extreme circumferences are seen; but P′EG, PEG are equal to S″P′E, S′PE respectively, the angles of deviation for the red and violet rays; hence, denoting the breadth by d, we have

$$d = 42^\circ\ 1'\ 40'' - 40^\circ\ 16'\ 20'' = 1^\circ\ 45'\ 20''.$$

In like manner, denoting the breadth of the secondary bow by d', and the distance between the bows by d'', we have

$$d' = 3^\circ\ 10'\ 48'',$$

$$d'' = 8^\circ\ 57'\ 10''.$$

These would be the dimensions and distance of the two bows, did the light emanate from a mere point; but the sun has an apparent magnitude of 32′. To apply the necessary correction, suppose the bows, as just determined, to be formed by the light proceeding from the highest point of the solar disk, and through E draw the straight line S$^{\text{v}}$H to

the lowest point of the disk. Then let SG revolve about E in the vertical plane SES^v, till it coincide with S^vH; all the lines of the figure, except S^vH, revolving with it, and retaining their relative positions. Any point, as I'', will describe an arc of 32′; and if we consider SG as constantly representing a beam of solar light, each of the small lines of homogeneous light determined by the diameter of the pupil, and making up the vertical linear spectrum, will be increased by an arc of the same length: consequently the breadth of each bow will be increased by 32′. We shall thus have for the true distance and breadth of the bows,

$$d = 2^\circ\, 17'\, 20'',$$

$$d' = 3^\circ\, 42'\, 48'',$$

and

$$d'' = 8^\circ\, 25'\, 10''.$$

These results of calculation have been verified by actual admeasurement.

The extent of the bow depends upon the position of the sun: when the sun is in the horizon, the arches are semicircles; as it rises they diminish, the primary bow ceasing to exist when its altitude is about 42°, the secondary when about 54°. When the sun is in or a little below the horizon, and the rain not distant, by giving to the observer a position sufficiently elevated, the top of a mountain, for example, a complete circle may be rendered visible.

When the rainbow is very brilliant, colored bands are sometimes seen within the primary, and also but more rarely without the secondary bow. They are explained by the wave theory.

115. *Halos.*

In northern latitudes, the sun and moon frequently appear surrounded with a luminous ring or *halo*, concentric with the body, and having an apparent semi-diameter varying from 22° to 23°. The lunar halo is simply a whitish band, occasionally fringed on the side next the moon with a pale red. The solar halo consists of bands of the prismatic colors, in which the hues, though far less vivid than those of the rainbow, are sufficiently distinct to be recognized. Its inner circumference, in color like that of the lunar halo of a pale red, is tolerably well defined; but the violet of the exterior ring gradually fades away, and blends with the general color of the sky.

A second halo, concentric with the first, and having an apparent semi-diameter of about 46°, is sometimes seen, occasionally at the same time with the first, but more frequently alone. It is less luminous than the former, and its colors are less distinct. At times the solar halos are accompanied by a horizontal circle of white light passing through the sun, and by portions of two inverted colored arcs touching the halos at their highest points. The points in which the horizontal circle and inverted

arcs intersect and touch the halos, are usually distinguished by bright spots or *mock suns*, called *parhelia.* Mock moons, called *paraselenæ*, are sometimes seen near the moon under similar circumstances.

The appearance of the two halos is satisfactorily accounted for, by the refraction of the solar rays, in their passage through minute crystals of ice supposed to be floating in the air. It is well known that ice frequently crystallizes in triangular prisms, in which the bases are perpendicular to the axes, and the rectangular faces are inclined to each other at an angle of 60°. These crystals being formed under certain conditions in the higher regions of the atmosphere, and lying in all possible directions, we can readily conceive that a great number of them will always have their axes perpendicular to the plane passing through the eye of the observer and the centre of the sun, whatever angle it make with the horizon. We can also conceive, that of these, many will be inclined to the solar beam at angles very nearly equal to the angle of inclination at which the deviation is a minimum, so that the light will emerge from them in rays sensibly parallel to each other, and hence capable of producing vision at a distance.

Now let ABC [Fig. 93] be a principal section of one of these prisms of ice by a plane passing through SE, the line drawn through the centre of the sun and the eye of the observer, and let S′I be a small

beam of solar light incident at such an angle that the deviation GOE of the mean rays shall be a minimum. The refracting angle of the prism being denoted by G, the deviation by D, and the index of refraction of ice for the mean rays by n, we have [Art. 48],

$$\frac{\sin \frac{D+G}{2}}{\sin \frac{G}{2}} = n.$$

In the present case, $n = 1.31$ and $G = 60°$; hence we have

$$sin \frac{(D+60)}{2} = 1.31 \times .5;$$

$$\tfrac{1}{2}D + 30° = 40° \; 55' \; 10'',$$

$$\tfrac{1}{2}D = 10° \; 55' \; 10'',$$

and $$D = 21° \; 50' \; 20''.$$

Hence $$SEO = D = 21° \; 50' \; 20''.$$

It thus appears that the halo which would be formed under the above conditions would have a semi-diameter of about 22°. But this is also the magnitude of the semi-diameter of the inner halo as determined by observation.

The above hypothesis, therefore, accounts for the formation of the inner halo. With respect to the order of the colors, it will be perceived that the red rays, suffering the least deviation, will form the inner circumference.

The halo of 46° may be accounted for by supposing the light which forms it to be refracted by those prisms which lie in such positions that the refracting angles are the right angles which the lateral faces make with the bases; for the minimum deviation by a prism of a refracting angle of 90°, is very nearly 46°.

116. *Coronæ.*

The small colored circles, only a few degrees in diameter, which are sometimes seen around the sun and moon, are called *coronæ.* They differ from halos in the order of the colors, having the violet within, the red without.

117. *Mirage.*

In certain states of the atmosphere, light is so modified in passing through it, that distant bodies appear multiplied and variously changed in form and position. Thus an inverted image of a ship just visible in the horizon, is sometimes seen beneath the ship itself, and sometimes suspended in the air directly above it. Sometimes this inverted image is visible when the real ship is actually below the horizon; and occasionally a second extraordinary image is perceived, erect and in contact with the first. These images are usually more or less distorted.

This phenomenon is called mirage. Its appearance is not confined to the vicinity of water. On the sandy plains of Lower Egypt it is almost of

daily occurrence, and assumes a peculiar character. On these plains, which are nearly level, and extend on all sides to the horizon, the only objects visible are a few villages, built in elevated positions to secure their inhabitants from the inundations of the Nile. In the morning and evening, the scene is such as the actual nature and arrangement of the objects should present; but in the middle of the day, when the earth is heated by an intense sun, a mirage of an extraordinary character is developed. The plain, at the distance of about a league from the observer, seems terminated by a general flood; the villages situated at a greater distance appearing like islands in the midst of an immense lake, each having beneath it an inverted image, such as would be formed by reflection from the surface of water. As the observer advances, the limits of the apparent inundation recede before him; and the phenomenon, ceasing for nearer objects, is reproduced with respect to others more remote. MONGE, who accompanied BONAPARTE to Egypt, and frequently observed this mirage during the march of the French army from Alexandria to Cairo, has given the following explanation of it:

The dry and sandy soil of the desert becoming intensely heated by the mid-day sun, the air in contact with the surface AB [Fig. 94] is rarified, and ascending currents are produced. A state of the atmosphere is thus determined, in which the strata, at first exceedingly rare, rapidly *increase* in

density from the earth upwards to a certain height *mn*, usually very small, above which they follow the ordinary law. This being the case, it is evident that a ray of light proceeding from an object G above CD (the stratum of maximum density), may fall so obliquely upon the strata immediately below CD as to be totally reflected at some point Q, and reach the eye of an observer situated at E, a point also above CD. Another ray will evidently reach E by the path GE, through a stratum of almost uniform density. Consequently the observer at E will see, at the same time, the object itself and an image of it: the one at G, nearly in its true position; the other in the direction of the tangent EG′, and depressed.

The reflection of pencils proceeding from different points of the object may be supposed to take place sensibly, if not rigorously, at the same surface, and hence the depressed image will be inverted. In effect, therefore, we may consider the plain as covered by a vast mirror; so that of all objects beyond a certain limit, and somewhat elevated, as the villages and lower parts of the sky, erect and inverted images will be seen at the same time. To the inverted images the rapid currents of air will impart a tremulous motion, such as is observed in the case of reflection from water slightly agitated. But will not the surface of the earth beneath this virtual mirror be seen at the same time, and render the

images confused? A little consideration will make it apparent that these parts of the plain will not be visible, the rays emanating from them being refracted upwards so as entirely to escape the eye of the observer. Thus, in all its essential circumstances, this singular illusion will bear a close resemblance to the reflection of the images of surrounding objects by an expanse of water. The distance from the observer, at which the phenomenon commences, will evidently depend chiefly upon the height of the stratum of maximum density, and upon the density of the strata between it and the surface of the plain.

The principle of the preceding explanation, viz. the reflection of light by a surface separating two strata of air of different densities, appears to be applicable to nearly all cases of mirage.

When the inverted image is elevated, the density of the air from the surface upwards must be conceived to decrease *very rapidly* for a short distance; its density at the surface being supposed greater than the normal density. This case is illustrated by figure 95, in which EG represents the surface of the earth or water, RS a stratum of the usual density at the height *mn*, above which the density varies according to the ordinary law, and GQE a ray of light emanating from an object G, suffering reflection at Q, and reaching the eye of the observer at E in the direction G′E.

The occasional existence of causes capable of producing, in the lower strata of the atmosphere, this *rapid* decrease of density, is not improbable.

Cases of lateral mirage, in which the extraordinary image is seen on one side of the object, instead of above or below it, have been observed. To explain these, a condition of the atmosphere must be admitted, in which strata of air of different densities are, for a small distance from the surface of the earth, in a position nearly vertical. Such a condition must evidently be very rare: it may, however, be supposed occasionally to occur in situations where a lake or plain is terminated on the east by a high mountain, and thus receives on one part the direct rays of the morning sun, while the adjacent part is in the shade. Figure 95 will illustrate the case of lateral mirage, if we suppose the plane EGSR to be horizontal instead of vertical.

NOTE.

In article 112 we have admitted the existence of an expression for the deviation in terms of the angle of incidence. We proceed to find this expression.

The cases upon which the explanation of the rainbow depends are presented in figures 96 and 97, in which the circles represent sections of drops of water, by planes passing through their centres and the line supposed to be drawn through the centre of the sun and the eye of the observer; and SI, SI, beams of solar light, which enter the drops at I, I; are reflected once in the first drop, and twice in the second; and finally emerge at I″ and I‴, in the directions I″E, I‴E. In the first case, the incident and emergent rays meet behind the drop; and in the second case, in front of it: the total deviation being, in the one case, equal to $180° - IPI''$; in the other, equal to $180 + IPI'''$. In the text, we have called the variable angle IPI″ or IPI‴ the deviation; and as it is the only part of the expression for the total deviation with which we are concerned, no ambiguity can result from continuing to designate it by that term. Referring, as we have already done, to both figures at the same time, since the angles which the chords II′, I′I″, etc., make with the radii drawn to their extremities I, I′, I″, etc., are all equal to each other, the angle CII′, the first angle of refraction, is equal to CI″I′ or CI‴I″, the last angle of incidence; and hence the angle SIR, at which the light falls upon the drop, is equal to EI″R′ or EI‴R′, the angle at which it emerges from it. Consequently the straight lines PC, PC, bisect the angles of deviation; and we have, in the one case, $IPC = I''PC$; in the other, $IPC = I'''PC$. Now in both cases we have

$$IPC = 180° - ICP - PIC.$$

But denoting the number of internal reflections by p, we have,

figure 96, $$ICP = \frac{\text{arc } II'I''}{2} = \frac{(p+1)\,ICI'}{2};$$

and fig. 97, $$ICP = \frac{\text{arc } IBI'''}{2} = \frac{2\,.\,180° - (p+1)\,ICI'}{2}.$$

We also have $ICI' = 180° - 2\,CII'$.

Hence, observing that in figure 97 we have

$$PIC = 180° - PIR,$$

we find, by substitution and reduction,

figure 96,

$$2\,IPC = (p+1)\,2\,CII' - 2\,PIC - (p-1)\,180°,$$

and figure 97,

$$\begin{aligned} 2\,IPC &= -(p+1)\,2\,CII' + 2\,PIR + (p-1)\,180° \\ &= -\left((p+1)\,2\,CII' - 2\,PIR - (p-1)\,180°\right); \end{aligned}$$

and denoting the angles of deviation, incidence and refraction, as usual, by D, I and R respectively, we get

$$D = \pm\left((p+1)\,2\,R - 2\,I - (p-1)\,180°\right)\ldots\ldots[J]$$

We also have

$$\sin I = n \sin R,$$

n being the index of refraction between air and water; and hence

$$\sin R = \frac{\sin I}{n}.$$

We may therefore consider D as found in terms of the angle of incidence.

If now for rays of any of the primary colors, the red, for example, we should calculate numerically the values of R for a series of values of I from 0 to 90°, and then by means of equation [J] determine the corresponding values of D, it would be found, as stated in article 112, that for one internal reflection, D would increase with I till the corresponding values were about 42° and $59\frac{1}{2}°$; after which, it would diminish with a further increase of

that angle; and that for angles of incidence differing little from $59\frac{1}{2}°$, the deviations would be sensibly equal to the maximum. In this manner, it would be possible to obtain the results given in the text for the red and violet rays, and for one and two interior reflections; but the process would be laborious in the extreme. The following method is the one actually employed.

Equation [J] gives by differentiation,

$$\frac{dD}{dI} = \pm\left(2\,(p+1)\,\frac{dR}{dI} - 2\right);$$

and equation $\sin I = n \sin R$, gives

$$\frac{dR}{dI} = \frac{\cos I}{n \cos R}.$$

Hence, by substitution, we get

$$\frac{dD}{dI} = \pm\left(\frac{2\,(p+1)}{n}\,\frac{\cos I}{\cos R} - 2\right) \text{.............} [K]$$

Putting this equal to zero, we have

$$(p+1)\cos I = n \cos R,$$

or $$(p+1)^2 \cos^2 I = n^2 \cos^2 R;$$

but $$\sin^2 I = n^2 \sin^2 R;$$

hence $(p+1)^2 \cos^2 I + \sin^2 I = n^2$,

or $(p+1)^2 \cos^2 I + 1 - \cos^2 I = n^2$;

and developing and reducing,

$$\cos I = \pm\sqrt{\frac{n^2-1}{p^2+2p}} \text{.................} [L]$$

To find the values of I, R and D, in any particular case, we substitute in equation [L] the proper numerical values of n and p; the value of I, thus obtained, gives, by substitution in equation $\sin I = n \sin R$, the corresponding value of R; and I and R, substituted in equation [J], determine the required value of D.

To complete the solution, we differentiate equation [K]. This gives

$$\frac{d^2D}{dI} = \pm \frac{2(p+1)}{n} \frac{(-\sin I \,.\, dI \,.\, \cos R + \sin R \,.\, dR \,.\, \cos I)}{\cos^2 R},$$

or

$$\frac{d^2D}{dI^2} = \pm \frac{2(p+1)}{n} \frac{1}{\cos^2 R} \left(-\sin I \,.\, \cos R + \sin R \frac{\cos^2 I}{n \cos R}\right) \text{[M]}$$

Now since $I > R$, and each is less than 90°,

we have $\sin I > \sin R$ ....[1]

and $\cos R > \cos I$....................[2]

We also have $n > 1$.......................[3]

Combining the 2d and 3d of these inequalities, we get

$$n \cos R > \cos I,$$

or

$$1 > \frac{\cos I}{n \cos R};$$

and combining this with the 1st and 2d, we get

$$\sin I \cos R > \frac{\sin R \cos^2 I}{n \cos R}.$$

The factor within the vinculum in the second member of equation [M], is therefore always negative; and the value of cos I, given by equation [L], corresponds to a maximum or minimum value of D, according as the expression for D given by equation [J] is affected with the sign + or —; that is, according as the incident and emergent rays meet behind the drop, or in front of it.

CHAPTER VI.

THE WAVE THEORY.

118. Observations upon the eclipses of Jupiter's satellites show that light is transmitted from one point of space to another with a finite velocity. This transmission may be supposed to be effected either by the actual translation of material particles from the luminous body to the point illuminated; or by the communication of motion from the one to the other, by means of the vibrations of an intervening medium. The former was the opinion of Newton; the latter, of Huygens. The theory of Huygens, for a long time neglected, was revived by Dr. Young; and, as developed by the labors of himself, Fresnel and others, is now almost universally adopted. It is called the *Theory of undulations*, or the *Wave Theory.*

The wave theory requires the admission of the following hypotheses:

1°. That an exceedingly rare and elastic fluid, or *ether*, as it is called, fills all space, even the interstices of all other material substances. In void space the distribution of the ether is supposed to be uniform, and its elasticity to be the same in all directions. The same is also supposed to be its

mode of existence in homogeneous uncrystallized matter, and in any crystal of which the primitive form is a regular polyhedron.

In different substances, the elasticity is supposed to vary with the nature of the substance: the same is supposed true of its density. In crystals of which the primitive form is not a regular polyhedron, the elasticity, in the same crystal, is supposed to vary with the direction.

2°. That the particles of a luminous body are in a state of rapid vibration, and possess the property of communicating similar vibrations to the molecules of the ether; and that the vibratory motions thus originated are propagated throughout the ether, from one molecule to another, in all directions, in a manner resembling that in which vibratory motions are propagated in other elastic fluids.*

* It is an inference, both from theoretical considerations and from actual experiment, that when an exceedingly small disturbance of molecular equilibrium occurs in matter, in any of its forms, each disturbed particle will be urged back to its place of rest with a force proportional to its distance from that point. When the trajectory is sought which a particle thus acted upon will describe, it is found that in general it will be an ellipse having its centre at the point of rest. It is also found that the period of revolution of the particle will be the same, whatever the extent of the orbit. In the extreme cases in which either the axes are equal to each other, or one of them is reduced to zero, the ellipse becomes either a circle or a straight line; and in the latter case, the revolution is changed into a vibratory motion, in which the times of vibration are independent of the amplitude.

3°. That when these vibrations reach the retina, through the medium of the ether within the eye, their action upon that membrane produces in us the sensation of light; in a manner analogous to that in which the vibrations of the air, striking upon the tympanum of the ear, produce the sensation of sound.

4°. That the color of the light is determined by the time of vibration, and its intensity or brightness by the absolute or maximum velocity of vibration; just as in the analogous case of the production of sound, the pitch or tone is determined by the time of vibration of the particles of air, and the intensity or loudness by their absolute velocity.*

In applying the conception of motion to the explanation of the phenomena of light, the reasoning is the same, so far as we shall wish to extend the explanations, whether the motion be regarded as one of vibration or revolution. In our illustrations, we adopt the conception of a right-lined and vibratory motion as the simpler.

*To illustrate this postulate, imagine a cycloidal pendulum [Fig. 98] vibrating at a given place, in indefinitely small arcs. Then, by the time of its vibration, we mean the time during which it passes from *a* to *b*, and from *b* back to *a*; and by the absolute velocity, that which it has at the lowest point *c*, evidently its maximum velocity. Now as in such a pendulum all the vibrations are executed in the same time, whatever the absolute velocity, or the amplitude of vibration to which that velocity is proportional, if we suppose it to vibrate with different absolute velocities, it may serve to illustrate the vibratory motions of the ethereal molecules in the case of homogeneous light of different intensities.

119. These hypotheses being admitted, we proceed to deduce from them, in a very elementary manner, some of their more obvious consequences.

Suppose, then, the ether to be diffused throughout a vacuum, or a homogeneous uncrystallized medium, and its equilibrium to be disturbed by the action of a luminous point; and to give simplicity to our conceptions, suppose also that the atoms of the point perform their vibrations in the same time, so that the light which emanates from it is perfectly homogeneous.*

Now it is a law derived from the mathematical investigation of vibratory motions, that, in such a case as we have just supposed, vibrations of all intensities are propagated with the same rapidity; the velocity depending only upon the elasticity and density of the fluid, and being therefore constant. Hence it immediately follows, that the vibrations will be propagated from the luminous point with equal velocities in all directions; and that, at any moment, the molecules which the disturbance has just reached will be situated on the surface of a sphere, of which the luminous point is the centre.

As the time of vibration changes with the intensity of gravity, to illustrate the case of homogeneous light of a different color, we have only to suppose the pendulum to vibrate on some other parallel of latitude.

*Since when the vibrations of the atoms are made in equal times, we must suppose that the resulting vibrations of the molecules are also made in equal times.

In order to a clearer conception of the manner in which the vibratory motions are propagated, let us now confine our attention to a single row of the ethereal molecules, supposed to be uniformly distributed on any radius of the sphere. Let aw [Fig. 99] be this radius; and suppose the action of the luminous atom, which determines the isochronous vibrations of the molecules of the row, to be such as to cause the first molecule to vibrate from a its position of equilibrium, to a', from a' to a'', and from a'' back to a, at right angles to aw, in a manner similar to that in which a pendulum, describing indefinitely small arcs, would vibrate about a the middle of $a'a''$, as its point of rest; so that at equal distances from a, the molecule has equal velocities in vibrating both towards and from a.

Again, let the ether be supposed to be so constituted, that this motion of the first molecule must cause a similar motion in the adjacent molecule x; the motion of x, a like motion in x'; and so throughout the row, the rapidity with which the motion is communicated being the same as that with which light is transmitted.*

* The phenomena of polarized light require us to assume that the vibrations of the ethereal molecules are performed in planes transverse to the direction in which the light is propagated. An example of this mode of propagation is seen when a stretched cord is struck at one of its extremities, in a direction perpendicular to its length.

Then if t be the time of a complete vibration of the first molecule, that is, the time of its passage from a to a', from a' to a'', and from a'' back to a; and if we also suppose that at the instant the molecule has reached a', the disturbance in the direction aw has just reached the molecule at b, it is evident that at the end of the time $\frac{t}{4}$ the molecules between a and b will be found in some curve a'------b; the first molecule having reached the point where its velocity is zero, and the molecule at b just beginning to move.

From similar considerations it may be shown that at the end of the several periods $2\frac{t}{4}$, $3\frac{t}{4}$, $4\frac{t}{4}$ or t, the positions of the molecules will be as represented in the figures 100, 101 and 102 respectively; the disturbance at the end of the time t having reached the molecule at e.

The velocity with which the disturbance is transmitted being constant, the distances ab, bc, cd, de, will be equal to each other. The directions in which the molecules are moving, are indicated in the figures by arrows. As we suppose the successive motions of all the molecules to be exactly similar to the motions of the first, equal velocities will correspond to equal displacements both above and below aw. Thus in consequence of the actual motions of the molecules at right angles to aw, the ether will be thrown into the form of a series of undulations or waves, which will be produced and

propagated from *a* towards *w*, as long as the disturbing cause, the vibratory motion of the luminous atom, continues to act.

It is important to observe, that the motions attributed to the ethereal molecules are such as may be considered infinitely small, the displacement of a molecule being supposed to remove it only an infinitely small distance from its position of equilibrium, and not to change the order of its position relative to the surrounding molecules.

120. *Phase.*

The term *phase* is used to denote the condition of a molecule with respect to motion and displacement. Two molecules are said to be in similar phases of vibration, when their motions and their displacements referred to the line *aw* are equal, and in the same sense; as [Fig. 102′] the molecules at a and e, s and s^{IV}, b' and f'.

When their motions and displacements are equal and in opposite senses, the molecules are said to be in opposite phases; as s and s', b' and d', c and e.

121. *Wave. Length of a wave.*

All the molecules included between any molecule of the row and the next in succession which is in a similar phase of vibration, constitute an ethereal wave or undulation; as, for example, those between a and e, s and s^{IV}, b' and f'. The distance between the two, is called the *length of the wave.* Thus the length of the wave *ace* is *ae;* that of the wave $b'cef'$,

is bf. The length of a wave is evidently the linear space through which the motion is propagated during the time t of a vibration. An inspection of the figure shows that a wave comprises molecules in every phase of vibration.

122. If the velocity of light be denoted by v, the time of vibration of a molecule by t, and the length of the corresponding wave by l, we shall evidently have

$$l = vt.$$

Thus light of each color has a length of wave peculiar to itself. For light of a given color, the value of t is the same in all media, and is not altered by the reflection or refraction of the rays. But since v depends upon the density and elasticity of the ether, and these are supposed to be different in different media, the velocity of the light, and consequently the length of the wave, will vary with the medium.

In the latter part of this chapter, it will be shown that the velocity of light, in material media at least, must be supposed to vary with the color; so that in passing from one kind of light to another, we must consider v variable as well as t. For the sake of simplicity, we shall continue to suppose the light homogeneous, except when the contrary is expressly stated.

During the combustion or incandescence of a body, its atoms must evidently be subjected to

forces indefinitely varied. Since then we suppose the motions of the ethereal molecules to be regulated by those of the atoms of the body, we must in general suppose the times and velocities of vibration of the molecules to be also indefinitely varied. The variations of the times producing waves of different length, between those of greatest and least length there will consequently be a vast number of intermediate lengths. We suppose them, however, to be reduced to seven different classes or species, corresponding to the seven primary colors of the solar spectrum.

Besides the undulations which produce the sensation of vision, we must admit the existence of others corresponding to the invisible rays of the spectrum, and which become sensible only by their heating or chemical effects.

123. *Intensity.*

With respect to the velocity of vibration, and the consequent intensity of the light at different distances from the luminous point, we should infer from the circumstances of the case that they would be diminished by distance; and in the introduction, we have deduced from the theory of emission the well known law according to which the intensity is actually found by experiment to vary. This law may also be shown to be a consequence of the wave theory.

Since the ethereal molecules constitute a perfectly elastic system, in the transmission of motion from one point of the system to another, the law of the *conservation of living forces* must be observed. Denote any two radii of the spherical wave by r and r', the surfaces of the corresponding spheres by s and s', and the velocities of the molecules at the surfaces by v and v'; then by this law we have

$$sv^2 = s'v'^2,$$

or $$s : s' :: v'^2 : v^2.$$

We also have

$$s : s' :: r^2 : r'^2;$$

and hence $$v^2 : v'^2 :: r'^2 : r^2,$$

or $$v : v' :: r' : r;$$

that is, at any two points of a luminous ray, the absolute velocities of the molecules are inversely as their distances from the origin of the disturbance.

As to the relation between the velocity and the impression upon the retina, or the intensity, we assume as an ultimate fact, indicated by certain analogies and satisfying the required conditions, that the latter is proportional to the square of the former. This, indicating the intensities by i and i' respectively, gives

$$i : i' :: v^2 : v'^2;$$

consequently we have

$$i : i' :: r'^2 : r^2,$$

the known law.

124. *Amplitude of vibration.*

By the amplitude of vibration, we mean the distance between the points at which the velocities of the molecules are zero; thus, the amplitude of vibration of the molecule at a is aa''. [Fig. 102.] This quantity is proportional to the absolute velocity of the molecule; and hence we may consider the intensity of the light as determined by the amplitude, the intensities in any two cases being directly as the squares of the amplitudes.

125. *Principle of the superposition of small motions.*

Before we proceed to the application of the preceding principles, we must consider a difficulty that will be likely to present itself to the mind of the student, on a fundamental point of the theory.

It may readily be admitted that light from a single luminous point is propagated as we have supposed; but how, it may be asked, can light, emanating from a number of different points, be propagated by the same mass of ether, at the same time, so as to produce distinct impressions upon the eye? Can these numerous systems of undulations exist in the ether simultaneously, without mutual disturbance? Improbable as the co-existence of undulations in the same mass of fluid may at first appear, it is evidently true in the case of the transmission of sound by air; for we can hear, with perfect distinctness, sounds produced by different sonorous bodies at the same time. It is also exem-

plified when the surface of water, or other liquid, is agitated at several different points at the same time; the systems of waves emanating from the different points crossing each other in all directions on the surface, and yet each system retaining its existence independently of all the others.

This principle, thus shown to be true inductively, is susceptible of mathematical demonstration. It is known in mechanics as the principle of the *superposition of small motions*, and may be thus stated: When the equilibrium of a system of material points is disturbed by forces which produce only such motions as may be considered infinitely small, these forces, however numerous, have their full effect without interfering with each other; that is, the effect upon the points, of any one of the forces, is the same as if the others did not exist. Hence it immediately follows, that we may apply to this case the ordinary rules for the composition and decomposition of forces. Thus, in the last illustration, for example, at any instant, the elevation of the water at each point, referred to a horizontal plane, is the algebraic sum of the elevations and depressions due to the waves separately considered.

126. *Interference of light.*

Similar systems of waves, are systems in which the amplitudes and times of vibration are equal. Let δ and δ' [Fig. 103] be two such systems, originating at the same time at the points a and b very

near each other, and propagated in parallel directions and in the same sense. A mere inspection of the figure shows that the corresponding molecules, as those at o' and o'', for example, are in similar phases of vibration. If then these systems become equally inclined towards each other, so that o' and o'' may coincide at some point o (in which case the distances ao and bo will be equal to each other), it is evident that they will tend to impress upon the molecule at o equal velocities in very nearly the same direction, so that [Art. 125] the resultant velocity may be considered equal to the sum of the components, and consequently the amplitude of vibration will be doubled, and the intensity of the light quadrupled. The same will evidently be true if one of the systems originate at d, e------, nearer o by one, two, or any number of entire undulations, or any even number of semi-undulations.

Again, if one of the systems as (δ') [Fig. 104] be moved in the direction in which it is propagated, through a space equal to half the length of a wave, it will be perceived that the corresponding molecules, as those at o' and o'', for example, will then be in opposite phases of vibration: Hence if these systems, their origin remaining fixed at a and b, be inclined so that o' and o'' may coincide at the same point o, they will tend to impress upon the molecule at that point equal velocities in very nearly opposite directions, and the resultant velocity may

be considered equal to zero: consequently the molecule will remain at rest, and the concourse of the two rays will produce darkness. The same will evidently be true if the distance of o from the origin of δ' be less than its distance from the origin of δ, by $3\frac{l}{2}$, $5\frac{l}{2}$, or any odd number of semi-undulations.

According to the theory, then, when two similar systems of waves intersect each other at a very small angle, the effect of their mutual action upon the molecule at their intersection is such as to quadruple the intensity of the light, when their paths, reckoning from their origins, are equal, or differ by an even number of semi-undulations; and to reduce it to zero, when the difference of the paths is equal to a semi-undulation, or to any odd number of semi-undulations. When the difference of the paths is intermediate, the resultant velocity will evidently be greater or less, according as the difference approximates to one or the other of the two limits.

This singular consequence of the theory, that when light is added to light, the result may vary with certain circumstances, so that in the one extreme case the intensity may be quadrupled, in the other annihilated, is called *the principle of interferences.* This principle was verified by Fresnel, in the following manner: A pencil of light, diverging from the focus of a lens of short focal distance, was

received upon two small metallic mirrors inclined to each other at a very obtuse angle. Two reflected pencils were thus obtained, intersecting each other in space at a very small angle, and, therefore, according to the theory, in a condition to exhibit the phenomenon of interference. The results were in accordance with the theory. But this being a fundamental principle, the evidence of its truth should be fully and clearly exhibited.

Let then S [Fig. 105] be the focus of the lens, and MN and MN′ the intersections of the mirrors by a plane passing through S, and perpendicular to their common intersection. Determine the virtual foci I and I′ [Art. 23]; and through the vertex of the isosceles triangle formed by joining the points M, I and I′, draw the line EMO perpendicular to the base; draw also EI, EI′, SG, SG′. Lastly, let KK′ be the section of a screen perpendicular to OE. Let us now see what phenomena should be presented by the light which falls upon the screen according to the principle of interferences. Since $SG = GI$ and $SG' = G'I'$, we have $SG + GE = EI$ and $SG' + G'E = EI'$, and hence $SG + GE = SG' + G'E$; and consequently the waves which are propagated in the directions SGE, SG′E traverse equal paths to reach the point E, and the intensity of the light at that point should therefore be a maximum. For any other point of the screen, as P, for example, the lengths of the paths PI′, PI

will differ by some quantity *d*. If this difference be equal to the length of a semi-undulation, or of any odd multiple of a semi-undulation, the intensity of the light at P should be a minimum; but if *d* be equal to any even multiple of a semi-undulation, the intensity at P should be a maximum. For any intermediate value of *d*, the intensity of the light at P should be greater or less, according as *d* approximates to an even or odd number of semi-undulations.

All these consequences of the theory are exactly verified in the experiment. Thus if we employ red light, tolerably homogeneous, we shall perceive upon the screen a number of narrow stripes, or fringes, alternately red and dark, in which the intensity of the light gradually decreases from the middle line of each bright stripe, to the middle lines of the adjacent dark stripes. These lines of maximum and minimum illumination are disposed parallel to the line of intersection of the mirrors, that is, perpendicular to the plane of the paper; that passing through E being bright, and the others symmetrically situated on each side of it. If the differences of the routes described by the rays which meet in these lines be calculated, they will be found to agree with the theory. Thus calling *l* the difference of the routes described by the rays which intersect in the middle of either of the bright stripes next to the central one at E, the

middle points of the other bright stripes will correspond successively to the differences $2l$, $3l$, $4l$, etc.; while the middle points of the dark stripes, from the first to the most remote, correspond to differences expressed by $\frac{l}{2}$, $3\frac{l}{2}$, $5\frac{l}{2}$, $7\frac{l}{2}$, etc.

That the preceding phenomenon is due to the mutual action of the rays of the two reflected pencils, may be shown by covering one of the mirrors, or intercepting the light reflected from it before it falls upon the screen; for, in that case, the stripes entirely disappear.

127. If we employ successively light of each of the primary colors, we shall obtain results which will differ from the preceding only in the positions of the corresponding stripes. They will vary with the color of the light; the violet stripes being least distant from E, the red stripes most distant.

128. When, instead of homogeneous light, we use that of the solar beam, the result is what we should expect from the superposition of the systems of stripes due to all the primary colors; thus, the middle stripe is white, while on each side of it are several iris-like stripes, the colors of which, after the third stripe, rapidly diminish in intensity, and, after the eighth, entirely disappear.

129. When the light employed is as we have supposed it, in article 126, such as is usually considered homogeneous, the middle stripe at E is very bright, and the adjacent dark stripes exhibit almost a total

absence of light; but as the distance from E increases, the bright stripes appear less bright, and the dark ones less dark, till at a certain distance from that point the variations in intensity entirely disappear. The cause of this is the imperfect homogeneity of the light; for whatever precautions may be taken to render a pencil of light simple, it will always be composed of rays more or less heterogeneous, and will therefore comprise undulations of various lengths, so that there will be formed on the screen as many different systems of stripes as there are kinds of light in the pencil; but as the positions of the stripes will vary with the length of the wave, the corresponding stripes will not be exactly superimposed, and there will evidently result a coincidence more or less perfect of the bright stripes of one system with the dark stripes of another, the degree of coincidence being nothing at E, and gradually increasing from that point till at length it becomes sufficiently perfect to render the intensity sensibly uniform. The number of stripes usually visible, varies from twenty to thirty on each side.

This explanation of the disappearance of the stripes after so few alternations has been verified by Fizeau and Foucault in the following manner: An aperture, very narrow and elongated, was made in the screen in a direction parallel to the fringes. 17, The two pencils of solar light, reflected from the mirrors and passing through this aperture, were

received upon a lens at a distance from the screen equal to its principal focal distance, and were thus thrown into parallel beams. These beams, after transmission through one or more prims, were received upon a second lens. Two spectra were thus formed at its principal focal distance, which were sensibly superimposed, and in which the light at every point was sensibly homogeneous, so that the interference of any rays by the intersection of the two pencils at the aperture would become apparent in the resulting compound spectrum. It was found that when the aperture was directly opposite the white central fringe, the spectrum was a normal solar spectrum; that when the screen was moved so that the difference of the distances from the origin of the pencils to the aperture was equal to half a wave length of violet light of a certain refrangibility, this color disappeared in the spectrum; that as the motion of the screen was continued, a dark stripe appeared successively in each of the colored spaces proceeding from the violet to the red, and then re-appeared in the violet when the difference became equal to three semi-wave lengths of violet light. It was also found that beyond a certain position of the screen, the dark stripes appeared simultaneously in two or more colored spaces, the number of them increasing as it advanced from that position; a result obviously due to the fact that in these cases the difference of the paths con-

tained exactly an odd number of semi-wave lengths of the colors which disappeared.

The number of wave lengths in the difference of the paths which produces a stripe in any case, can be easily calculated. Let l and l' denote the wave lengths corresponding to any two fixed lines of the spectrum, d the difference of the paths, n and n' the number of times l and l' are contained in d, and m the number of dark stripes in the spectrum between the assumed fixed lines; then

$$d = nl \text{ and } d = n'l',$$

and hence $$nl = n'l'.$$

We also evidently have

$$n' = n + m;$$

whence we get

$$n = m\,\frac{l'}{l - l'} \text{ and } n' = m\,\frac{l}{l - l'}.$$

l and l' can be calculated as will be shown in a subsequent article, and m can be determined by counting the dark stripes between the fixed lines. In one experiment, in which m was 141, the fixed lines being E and F, n' was found to be 1737. In another experiment, the number of wave lengths corresponding to a given stripe was found to be more than 8000.

130. If a very thin plate m of any transparent substance, a film of mica, for example, be interposed in the path of one of the reflected pencils, the

$n = \frac{n'l'}{l}$ $\quad \frac{n'l'}{l} = n' - m$

$n = n' - m$ $\quad n'l' = n'l - ml$

$n'l - n'l' = ml$

$n' = \frac{ml}{l - l'}$

stripes will be shifted towards the side of the interposed plate; a result indicating that light moves more slowly in mica than in air. For if the velocity of light in mica be less than in air, the wave length in the one will be less than the wave length in the other [Art. 122], and hence the number of wave lengths in the thickness of the plate m of mica will be greater than in the thickness of an equal plate n of air; and if the difference be, for example, half an undulation, the effect will be the same as if the length of the path in which the plate is interposed was increased by $\frac{l}{2}$; so that E should in this case be a dark spot, and the bright spot corresponding to equal paths should be shifted towards the side of the retarded ray, and thus all the stripes be displaced, as they are actually found to be, in the same direction.

131. It is important to remark, that it is essential to the production of the stripes, that the pencils of light which we employ should have the same origin. The reason of this will appear, if we consider that in order to satisfy the conditions of interference, we must attribute to the two pencils a similarity which it is highly improbable that light emanating from two distinct sources should ever possess, at least for a period of sensible duration.

132. Since the waves may be considered as diverging from the virtual foci I and I′ [Fig. 106], if, about these points as centres, two series of equi-

distant arcs be described, separated from each other by an interval equal to the length of a semi-undulation, and those of which the radii contain an odd number of semi-undulations be dotted and the others represented by full lines, then it is evident that while the intersections of the full arcs by the dotted arcs determine points in the middle lines of the dark stripes, the intersections of the full arcs by full arcs, and of the dotted arcs by dotted arcs, determine points in the middle lines of the bright stripes. Of these points, those which belong to a stripe of the same order, whether bright or obscure, lie on the arc of a hyperbola, of which I and I′ are the foci and O the centre; for they are determined by the intersection of lines drawn from I and I′, which differ by the constant quantity $n\,\frac{l}{2}$. Thus, if we take, for example, the first dark stripe at the right of the central bright stripe, we have for the point x, $\mathrm{I}'x - \mathrm{I}x = \frac{l}{2}$; for x', $\mathrm{I}'x' - \mathrm{I}x' = \frac{l}{2}$; and so on for the remaining points x'', x''', etc. Admeasurements made by Fresnel have verified this deduction from the theory.

133. *Determination of the length of a wave.*

The actual length of an ethereal wave, is an element of the greatest importance. To show in what manner it may be determined, we employ the triangle Exa [Fig. 106]. This triangle, formed by the intersection of the full arc Ex, the adjacent dotted arc ax, and the straight line EI, may, on account of

the smallness of the arcs Ex, ax, be considered rectilinear. Also since the arcs Ex and ax are perpendicular to the radii I′E, IE respectively, the angle Exa is equal to the angle IEI′. Hence denoting IEI′ by i, the distance xz between the dark points immediately adjacent to the bright point E by f, and a semi-undulation as before by $\frac{l}{2}$, we have Ex sensibly equal to $\frac{1}{2}f$, Ea equal to $\frac{1}{2}l$; and, hence, by a principle of plane trigonometry,

$$\tfrac{1}{2}l = \tfrac{1}{2}f \sin i,$$

or

$$l = f \sin i;$$

that is, the length of a wave in terms of i and f, quantities which, though exceedingly small, may be measured with great exactness. Fresnel determined their values for red light, and found the corresponding value of l, that is, the length of a wave which produces the sensation of red light, to be 0.0000256 of an inch. This was found to agree with the results of certain admeasurements of Sir I. Newton, made without any reference to the wave theory, but which actually involved the determination of the lengths of ethereal waves for all the primary colors. These admeasurements, and their relation to the theory of undulations, will be fully made known in the next chapter. The lengths of the ethereal waves, as determined by Fraunhofer, are given in the following table. The velocity of light is supposed to be 192,700 miles in a second.

Designation of the ray.	Length of an undulation in parts of an inch.	Number of undulations in an inch.	Number of vibrations in a second.
Line B.........	.00002708	36918	451 millions of millions.
Line C.........	.00002583	38719	473 "
Middle red......	.00002441	40949	500 "
Line D.........	.00002319	43123	527 "
Middle orange...	.00002295	43567	532 "
Middle yellow...	.00002172	46034	562 "
Line E.........	.00002072	48286	590 "
Middle green....	.00002016	49609	606 "
Line F.........	.00001906	52479	641 "
Middle blue.....	.00001870	53472	653 "
Middle indigo...	.00001768	56569	691 "
Line G..... ...	.00001689	59205	723 "
Middle violet....	.00001665	60044	733 "
Line H.........	.00001547	64631	789 "

Explanation of reflection and refraction.

134. When a pencil of light falls upon the surface of a transparent medium,

1. It is divided into two portions, one of which is reflected and the other transmitted; and,

2. These portions are reflected and transmitted in directions which bear a constant relation to the direction of the incident light.

The division of the incident light into reflected and transmitted rays has been shown by POISSON to be a necessary consequence of the wave theory.*

* The mathematical laws which govern the propagation of vibratory motions in fluids, constituted as we have supposed the ether to be, have been investigated by POISSON, FRESNEL, CAUCHY,

Previously, however, Dr. YOUNG had given an illustration of it, based upon the fact that when one elastic ball strikes another of equal mass at rest, it imparts to the latter all its velocity; but that when it strikes a ball of greater or less mass than itself, it always retains more or less of its motion.

In this illustration, the action of two infinitely thin strata of the ethereal molecules in a homogeneous medium is supposed to be analogous to the collision of the balls when the masses are equal; each stratum of the ether, in its turn, communicating all its velocity to the adjacent stratum in front of it, so that no motion remains to be propagated backwards towards the origin of the vibrations, and consequently no reflection can take place.

In the case of two contiguous homogeneous media, in which the ether may be conceived to be of different densities, it is supposed that the action of the last stratum of the one upon the first stratum of the other is analogous to the collision of the balls when the masses are unequal; and that as the first ball does not communicate all its velocity to the second, so the last stratum of the first medium does not communicate all its velocity to the first stratum of the second medium. The velocity retained by

and others. Objections to the theory, apparently insurmountable, have thus been obviated; and, in some instances, fortunate conjectures, having, as in the present case, no other basis than analogy, have been elevated to the rank of demonstrable truths. We shall occasionally be obliged to refer to these conclusions of analysis.

the last stratum is then supposed to re-act upon the stratum immediately behind it, and create a vibration which is propagated backwards in the first medium. Thus, as a consequence of the division of the velocity of the last stratum of the first medium between this stratum and the first stratum of the second, two new systems of vibratory motions are originated at the common surface; one propagated forwards in the new medium, and constituting the refracted rays; the other backwards in the first medium, and constituting the reflected rays. The foregoing is indeed but an illustration, but it will at least serve to give the student precise ideas of the important phenomenon in question.

135. We have next to consider the *directions of the reflected rays*. Let AB [Fig. 107] be the reflecting surface, and FI and EI′ two incident rays or systems of undulations proceeding from the same luminous point, supposed to be at an infinite distance. These rays will be parallel to each other; and if from I, IC be drawn perpendicular to EI′, the molecules at I and C will evidently be in the same phase of vibration, and IC will represent a portion of the surface of the principal wave at the instant at which it meets the reflecting surface at I. Now it may be admitted as a mode of action of the ether, that the molecule at I, as soon as the wave reaches it, becomes a new centre of disturbance, from which proceed rays in all directions,

and of different intensities; and that the same is true of all the molecules in the reflecting surface, as they are successively reached by the different portions of the wave.

Of the reflected rays or systems of undulations, it is required to determine those which satisfy the conditions of visibility; that is, those which are in complete accordance.* Of the rays which emanate from the points I and I′, let us take the two which pass through a molecule M of the ether, so distant from AB that they may be considered parallel (in which case their intensities will evidently be equal), and which also make with AB the same angles as the incident rays FI and EI′. Let IK and I′L be these rays, and draw I′C′ perpendicular to IK. The triangles ICI′, IC′I′ will be equal, and we shall have CI′ and C′I equal to each other. Then since the molecules at I and C are in the same phase of vibration, and the undulations are transmitted with equal velocities along the lines CI′ and IC′, the molecules at I′ and C′ will also be in the same phase of vibration, and the undulations propagated in the directions IK and I′L will therefore be in complete accordance throughout, and will impress upon the molecule at M equal velocities in the same sense.

It thus appears that the reflected rays which are inclined to the surface at the same angle as the

* Two systems of undulations are said to be in *complete accordance*, when the corresponding molecules are in similar phases; in *complete discordance*, when they are in opposite phases.

incident rays, satisfy the required condition, and will affect the eye placed at M the point of concourse with a sensation of light in the direction of the points of incidence.

But it can be shown that this is not true of the reflected rays which are inclined to AB at any other angle than that of incidence. Let IK′ and I′L′ be two rays meeting at a point N, so distant that they may be considered parallel, but not making with the surface angles equal to EI′A; and draw I′C″ perpendicular to IK′. Now it is evident that we may suppose the point I to have been taken at such a distance from I′, that the difference of IC″ and I′C may be just equal to the length of a semi-undulation. This being admitted, the undulations propagated along the lines IK′, I′L′ will be in complete discordance; and since their intensities are equal, will, when they arrive at the point N, mutually destroy each other: no light will therefore be reflected in the direction of these rays. In this manner it may be shown that, in general, all the reflected rays, except those which obey the ordinary law, will destroy each other by interference.

136. It is evident, that if the ray IK′ were suppressed, the ray I′L′ would become visible; and hence that if, by any means, we could suppress a sufficient number of the elementary rays which destroy the corresponding rays reflected at I′, the light would appear to diverge from that point. In

verification of this, it is found that when light is reflected by a triangular mirror, of which one of the angles is exceedingly small, so that the reflecting surface is reduced almost to a line, the light reflected by the part of the mirror near the vertex of this angle no longer obeys the usual law, but forms a divergent pencil, the divergency being greater as the point of incidence is nearer the vertex.

137. *The directions of the refracted rays.*

Denoting the velocities of light in the two media by v and v', and the corresponding lengths of the waves by l and l'' we have [Art. 122]

$$l = vt, \text{ and } l' = v't;$$

and hence, $$l : l' :: v : v';$$

that is, the lengths of the waves in the two media are proportional to the velocities of light.

As in the preceding case, suppose the incident rays FI and EI′ [Fig. 108] to be parallel to each other. Of the rays which enter the new medium, diverging from I and I′, it is required to determine those which satisfy the conditions of visibility.

Let IK and I′L be two of these rays, which meet at a point so distant that they may be considered parallel to each other; and draw IC and I′M perpendicular respectively to I′E and IK. Then since the undulations propagated in the directions FI and EI′ arrive at the points I and C at the same

time, and in the same phase of vibration, if we suppose the lines I′C and IM to be to each other as the velocities of light or the lengths of the waves in the two media, it is evident that the undulations propagated along these lines will reach M and I′ at the same instant and in the same phase, and consequently will proceed in the directions MK and I′L in complete accordance. Hence we have the proportion

$$l : l' :: I'C : IM;$$

but $$I'C = II' \cos II'C = II' \sin PI'E,$$

and $$IM = II' \sin II'M = II' \sin QI'L;$$

therefore

$$l : l' :: \sin PI'E : \sin QI'L;$$

or, denoting the angles of incidence and refraction by I and R,

$$\frac{\sin I}{\sin R} = \frac{l}{l'} = \frac{v}{v'} = n;$$

that is, the sines of the angles of incidence and refraction are in a constant ratio.

In the same manner as in the preceding case, it may be shown that the rays which do not satisfy this relation are destroyed by interference.

If we suppose the second medium to be more refractive than the first,

or $$I > R,$$

we shall have $$v > v'.$$

According to the wave theory, therefore, light moves with the greater velocity in the less refractive medium, as already found to be true in a particular case [Art. 130].

138. Since for light of a given color, the index of refraction is equal to the ratio of the lengths of the waves, or of the velocities of the light in the two media, if rays of all colors were propagated with equal velocities in the same medium, the index would be the same for all the transmitted rays, and there could be no dispersion. In order, therefore, to account for the separation of the refracted rays, we must admit that rays of different colors are transmitted with unequal velocities in the same material medium.

It will be perceived, that in this respect the analogy between light and sound fails; inasmuch as in the same medium, the velocity of propagation of sounds of all notes, high or low, is the same.

139. The existence of a relation between the velocity of propagation and the length of the wave in the case of light, and its absence in the case of sound, are both in accordance with the conclusions of analysis. When the problem of the velocity of wave propagation in a refracting medium is solved on the hypothesis that the distance between the molecules of the elastic fluid is infinitely small in comparison with the length of a wave, the velocity is found to be the same for waves of all lengths.

But in the more general case, in which the distance of the molecules and the length of a wave are regarded as magnitudes of the same order, the velocity is found to be different for waves of different lengths; the absolute values of the velocities in both cases being, of course, dependent upon the nature of the medium throughout which the ether is supposed to be diffused. The first solution answers to a case like that of the transmission of sound by the vibrations of air, in which the length of even the shortest wave is vastly greater than the distance between the molecules: the second solution is applicable to the case of light, in which the distances between the molecules and the lengths of the waves are both of the same order of magnitude.

140. The reflection of light is sometimes attended with a change of direction in the motion of the molecules of the reflected undulations. This change does not affect the general result, nor the reasoning upon which the preceding explanation of reflection is based; but, as in certain phenomena, to be considered hereafter, it will have to be taken into the account, it is proper that the student should be made acquainted with it.

A clear conception of this change, and its consequences, may be obtained by considering the manner in which the wave represented in figure 102 has been supposed to be formed, and observing that

if the molecule at a, after having made a number of complete vibrations, should, in vibrating from its position of equilibrium, instead of continuing to move downwards, have its motion reversed, and move upwards, the wave thus produced would be similar to that represented in the figure 109 γ', so that the corresponding molecules in the two waves would have equal velocities in opposite directions. The conditions of complete accordance and discordance in two systems of undulations, in which the vibrations should be thus in opposite directions, would evidently be directly the reverse of those given in article 126. An inspection of figures 109 and 110 will render this apparent.

141. This change of direction is indicated when we attempt to explain, by the theory of undulations, the production of color in certain cases, as, for example, in the formation of soap bubbles. Dr. Young illustrated it by the analogy of the elastic balls, and Poisson subsequently deduced it from a rigorous analysis. We have already given Dr. Young's illustration, as far as was necessary to elucidate the general fact of the division of the incident light into reflected and transmitted rays [Art. 134]. We now resume it.

In the collision of the elastic balls, we know that the direction of the impinging ball after the shock, depends upon its relative mass; moving in its original direction when its mass is greater than that of

the impinged ball, in the opposite direction when it is less. It may be supposed that in a manner somewhat analogous, the direction of the vibratory motions of the ether in the first medium, constituting the reflected undulations, depends upon the relative densities of the two media. Thus, admitting the density of the ether to be the greater in the denser medium, we may suppose that the molecules, after reflection, vibrate in the original direction when the first medium is more dense than the second, and in the opposite direction when less dense; that when, for example, the first medium is air, and the second glass, the direction of the vibratory motion is changed after reflection; but that when the first is glass, and the second air, the molecules, after reflection, vibrate in the original direction.

CHAPTER VII.

COLORS OF THIN PLATES.

142. All transparent substances, when reduced to plates of extreme thinness, exhibit the phenomenon of color. Familiar illustrations of this fact are seen in the changing hues of a polished metallic surface, as it becomes covered with a pellicle of oxide on exposure to heat; in the colors of soap bubbles, and the films obtained by dropping oil upon water; and in the fringes observed in crystals of mica, Iceland spar, sulphate of lime, and minerals of similar structure. That the colors are due to the thinness of the media, may be inferred from the phenomena of the soap bubble when protected from currents of air by a glass vessel. Thus, at first, the bubble appears uniformly white; but as the envelope becomes thinner by the gradual descent of its particles, colored rings become visible, which, commencing at the top where the thickness is least, and progressively dilating and moving downwards, present the appearance of a series of brilliant horizontal zones. When the top is reduced to a certain degree of thinness, it becomes black; after which the bubble soon bursts.

This subject was first investigated by Sir I. NEWTON. We shall give a brief account of his mode of proceeding, and the results at which he arrived.

When a plano-convex lens, of great focal length, is laid upon a plane glass, as represented in figure 111, a thin plate of air is formed, the thickness of which increases from the point of contact, and is the same at equal distances from that point. When this combination is placed before an open window, in the full light of day, and a slight pressure is applied to the lens, the eye of an observer, directed towards the plate, and situated at the point O, a little above it, and as nearly as possible in the axis of the lens, will perceive a number of concentric rings of vivid colors encircling a dark spot at the point of contact. If the eye be placed at O′ on the opposite side of the plate, another system of rings will be observed, less vivid than the former, and surrounding a central bright spot. The first system is evidently formed by the reflected rays of the incident beam; the second by the transmitted rays.

We shall first consider the rings formed by reflected light. These rings are so disposed as to form a series of circular spectra; each spectrum exhibiting a greater or less number of the prismatic colors, of different degrees of purity. The following table shows the order in which the colors succeed each other in the several spectra, beginning at the central black spot:

First spectrum, or order of colors	Black, very faint blue, brilliant white, yellow, orange, red.
Second spectrum.	Dark purple or rather violet, blue, green (very imperfect, a yellowish green), vivid yellow, crimson red.
Third spectrum.	Purple, blue, rich grass green, fine yellow, pink, crimson.
Fourth spectrum.	Green (dull and bluish), pale yellowish pink, red.
Fifth spectrum.	Pale bluish green, white, pink.
Sixth spectrum.	Pale bluish green, pale pink.
Seventh spectrum.	Very pale bluish green, very pale pink.

After the seventh, the colors are so faint as scarcely to be distinguished from white. The whole system of colors is called *Newton's scale;* and the colors are said to be of the first, second, third, etc. order, according to the number of the spectrum to which they belong. Thus, the pink of the third order, is the pink which is found in the third spectrum.

If the above experiment be repeated with homogeneous light, the red light of the solar spectrum, for example, the system of variously colored rings will be replaced by a series alternately red and dark; the central spot being black, and the adjacent ring red. The rings will also be more numerous than when white light is used; the number being larger, the greater the degree of homogeneity of the light.

It must not be supposed that the light of a bright ring is equally intense throughout, or that the dark rings are uniformly dark. On the contrary, from

the middle of each bright ring, at which the intensity is at its maximum, the illumination gradually diminishes on each side to the middle of the adjacent dark rings, where there is almost a total absence of light.

If light of each of the primary colors be successively thrown upon the lens, the rings alternately bright and dark will be found to vary in their dimensions with the color of the light employed; their breadth and diameter being least when violet light is used, and increasing in the order of the solar spectrum, to the red, in which they are greatest.

We should infer from the foregoing, that, in white light, the phenomenon would be similar to that which is actually presented. For since in consequence of the variation of the diameters, the rings simultaneously formed would not be exactly superimposed, it is evident that the colors would be variously blended, so as to produce rings of a great variety of tints, and, after a certain number of repetitions, a uniformly white surface. The explanation of the phenomenon, when the light is white, is thus reduced to that of the simpler one of the alternate bright and dark rings when the light is homogeneous. In what follows, we shall therefore suppose the light to be simple.*

* The disappearance of the rings, in all cases, after a certain number of alternations, is due to the imperfect homogeneity of the light; and also to the circumstance that the rings diminish in

Proceeding to ascertain the mathematical laws of the phenomenon, NEWTON measured the diameters of the rings* with great care; and from a comparison of them, deduced the following results:

1. That the squares of the diameters of the bright rings, taken in the order of succession from the centre, are as the odd numbers of the natural series,

1, 3, 5, 7, 9, etc.; and,

2. That the squares of the diameters of the dark rings are as the intermediate even numbers,

0, 2, 4, 6, 8, 10, etc.

Now the squares of the diameters of the rings are as the thicknesses of the plate of air, at the points where they are formed. For, denoting the radius of the curved surface of the lens by r, the thickness of the plate of air at any point I [Fig. 112] by e, and EI′ the radius of the corresponding ring by s, we have

$$s^2 = 2re - e^2;$$

or, since, e being very small, e^2 may be neglected,

$$s^2 = 2re;$$

and for any other ring, for which the radius and

breadth as they depart from the centre, so that were the light strictly homogeneous, they would, at a certain limit, become too close and fine to be seen.

* Employing the term *diameters of the rings*, to denote the diameters of the circles corresponding to the brightest parts of the colored rings, and the darkest parts of the obscure rings.

thickness are denoted by s' and e' respectively, we have

$$s'^2 = 2\,re'.$$

Consequently we have

$$s^2 : s'^2 :: e : e'.$$

From this it immediately follows, that the thicknesses of the plate at the points where the rings are formed, are, for the bright rings, as the numbers

1, 3, 5, 7, 9, etc.;

for the obscure rings, as the numbers

0, 2, 4, 6, 8, 10, etc.

The relative thicknesses being known, to determine their absolute values, it is only necessary to find the thickness for any one of the rings. This may be found by means of the equation

$$e = \frac{s^2}{2r}.$$

The value of r, for the lens which NEWTON used, was 91 inches. To obtain a value of s, he selected a dark ring, the fifth in order from the centre; and found its diameter measured on the upper surface of the lens, to be exactly one-fifth of an inch. The half of this number, corrected,* squared, and divided by 182, gave, for the thickness of the plate of

* Certain small corrections were applied for the refraction and thickness of the plate, and also for the slightly oblique position of the eye. Far more refined methods of admeasurement are now employed, in which the necessity of these corrections is avoided.

air at which this ring was formed, the $\frac{1}{17800}$ of an inch. Now as the thicknesses at which the consecutive rings are formed are as the numbers

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \text{etc.},$$

the thickness in any case at the fifth dark ring is ten times that at the first bright one; and hence in the case in question, the first bright ring was formed at the thickness of the $\frac{1}{178000}$ of an inch. Consequently the consecutive bright rings were formed at the thicknesses

$$\frac{1}{178000}, \frac{3}{178000}, \frac{5}{178000}, \frac{7}{178000}, \text{etc.},$$

and the dark rings at the thicknesses

$$\frac{2}{178000}, \frac{4}{178000}, \frac{6}{178000}, \frac{8}{178000}, \text{etc.}$$

These determinations belong to the rays of the most luminous part of the spectrum. As the diameters of the rings vary with the light employed, for rays of any other color, different values would evidently be found. NEWTON determined the values for rays of all the primary colors.

As yet we have supposed the space between the glasses to contain air only. If the experiment be varied by enclosing between them any other transparent fluid, or by rendering the interval a vacuum, it will be found that while the relations between the diameters of the rings remain the same, the absolute values of the diameters, and consequently the values of the corresponding thicknesses of the plates, vary, and in such a manner, that for rings of a given order, the thicknesses are inversely as

the indices of refraction of the substances composing the plates. Thus if a drop of water be introduced between the glasses, the rings will be seen to contract; and if their diameters in air and water be compared, it will be found that the corresponding thicknesses of the plates are as four to three; that is, in the inverse ratio of the indices of refraction.

The thicknesses above given were determined on the supposition that the eye was placed on the axis of the lens; or, in other words, that the rings were formed by the light which fell upon the lens at right angles. When the eye is placed without the axis, at different angular distances from it, and the rings are consequently formed by light which falls upon the lens at different obliquities, their diameters, the relations between them remaining constantly the same, will be seen to increase rapidly with the obliquity. It has been found that the thickness corresponding to a ring of a given order, seen at different obliquities, varies as the secant of the angle of refraction, or its equal the angle of incidence on the exterior medium. Thus the thickness for any ring at a perpendicular incidence being denoted by e, and the thickness for the ring of the same order when the angle of refraction is r, by e', we have

$$e' = e \sec r.*$$

* This law, supposed by NEWTON to be restricted to incidences below 60°, has recently been verified up to an incidence of 86° 14′, the greatest angle at which the rings could be distinguished.

143. *Rings formed by transmitted light.*

In this system of rings, when the light is homogeneous, the central spot is of the same color as the light employed; and any bright transmitted ring is at the same distance from the centre as the corresponding dark one of the reflected system.

When the light is white, the central spot is also white; and the color transmitted at any particular thickness of the plates, is complementary to that which is reflected at the same thickness. In all other respects, the rings observe the same laws as those formed by reflected light.

144. *Explanation of the preceding phenomena.*

The principle of interferences furnishes a simple and satisfactory explanation of the phenomena just described. We shall first consider the case in which the light, supposed to be homogeneous, falls upon the surface of the plate of air at right angles, or nearly so. Let LI [Fig. 111] be one of the rays of such a pencil, incident upon the first surface of the plate at I, and upon the second at I′; then there will be a partial reflection at both these points, and the point I may be regarded as the origin of two new systems of undulations, the one proceeding directly from I to L any point in its direction, the other from I to I′ and thence to L; the lengths of the paths by which they reach the point L differing by twice the thickness II′, or, denoting II′ by e, by $2e$.

Suppose now this space to be equal to half the length of a wave; that is, that we have

$$2\,e = \tfrac{1}{2}\,l, \quad \text{or } e = \tfrac{l}{4}.$$

Then since the reflection at I takes place in the denser, and at I′ in the rarer medium, and the velocities of vibration, after reflection, are consequently in opposite directions, it is evident that the case is identical with that of article 140; and that these two systems of reflected undulations will be in complete accordance, so that the eye placed at L will perceive a bright point in the direction LI′.

The same will evidently be true, if we have $2e$ equal to any odd multiple of $\frac{l}{2}$. It will therefore be true for all the values of e in the equations

$$2e = \tfrac{l}{2}, \quad 2e = 3.\tfrac{l}{2}, \quad 2e = 5.\tfrac{l}{2}, \quad 2e = 7.\tfrac{l}{2}, \text{ etc.};$$

or, $$e = 1.\tfrac{l}{4}, \quad e = 3.\tfrac{l}{4}, \quad e = 5.\tfrac{l}{4}, \quad e = 7.\tfrac{l}{4}, \text{ etc.}$$

But if we have $2e$ equal to $2\frac{l}{2}$, or any even multiple of $\frac{l}{2}$, then it is clear that the undulations will be in complete discordance; and, if their intensities be supposed equal, or nearly so, that the eye at L will perceive a dark point in the direction of the reflected rays. A succession of dark points will therefore correspond to all the values of e given by the equations

$$2e = o, \quad 2e = 2.\tfrac{l}{2}, \quad 2e = 4.\tfrac{l}{2}, \quad 2e = 6.\tfrac{l}{2}, \text{ etc.};$$

or, $$e = o, \quad e = 2.\tfrac{l}{4}, \quad e = 4.\tfrac{l}{4}, \quad e = 6.\tfrac{l}{4}, \text{ etc.}$$

With respect to the intensities of the two systems, as the amount of light reflected at I is small,

compared with that which is transmitted, it is evident that the latter system of undulations should be very nearly equal in intensity to the first.

It is thus shown to be a consequence of the theory, that the eye, placed at a point nearly on the axis of the lens, and a little above it, so as to receive the reflected rays, will perceive at the centre E a dark spot, surrounded by a system of rings alternately bright and dark, which not only present the same general appearance as those actually observed, but are formed at thicknesses which bear to each other the very relations derived from experiment.

Since in the preceding explanations we have assumed the thickness at which the first bright ring is formed, to be equal to one-fourth the length of a wave, the quantities obtained by multiplying this value of the thickness, for the rays of the primary colors, by four, should be identical with the lengths of the waves determined by the experiment of the mirrors. The comparison of the two sets of values was made by Fresnel, and found to be perfectly satisfactory.

The increase in the diameters of the rings, as we employ in succession the primary colors from the violet to the red, is evidently due to the corresponding changes in the lengths of the waves, the thicknesses of the plate at which the rings are formed being directly proportional to the lengths.

The relation between the thicknesses at which the corresponding rings of the same color are formed, in the case of plates of different substances, viz. that the thicknesses are inversely as the indices of refraction of the substances of the plates, may also be deduced from the theory. Thus denoting by l, l', and l'' the lengths of the waves in a vacuum and in any two refracting media respectively, and by n and n' the indices of refraction of the media, we have [Art. 137],

$$\frac{l}{l'} = n, \text{ and } \frac{l}{l''} = n';$$

and hence $$nl' = n'l'',$$

or $$l' : l'' :: n' : n;$$

that is, the lengths of the waves are inversely as the indices of refraction of the media. But denoting the thicknesses of the plates of the two substances, at the points at which the corresponding rings are formed, by e and e', we have

$$l' : l'' :: e : e':$$

consequently $$e : e' :: n' : n;$$

that is, the thicknesses of the plates are inversely as the indices of refraction of the substances composing them.

144'. We will now consider the general case in which the light falls upon the plate at any angle whatever. Let LI [Fig. 113] be a ray incident upon AB the first surface of the plate at I; IL',

II″ the corresponding reflected and refracted rays; and I″I′L″ the part of the latter ray which is reflected from the second surface CD at I″, and, emerging in the direction I′L″, interferes with the parallel reflected ray IL′. Draw I′P perpendicular to IL′, and denote by l' and l the lengths of the waves in glass and in air, and by p' and p the number of times $\frac{l'}{2}$ and $\frac{l}{2}$ are contained in IP and II″I′ respectively; then ~~while~~ the undulations propagated from I in the glass describe the space IP or $p' . \frac{l'}{2}$, the undulations propagated in the plate of air in the direction II″I′ pass over a space II″a equal to $p' . \frac{l}{2}$, and hence fall behind those propagated in the glass by a space aI′ equal to

$$\mathrm{II''I'} - \mathrm{II''}a = p . \tfrac{l}{2} - p' . \tfrac{l}{2} = (p - p') . \tfrac{l}{2};$$

and in order that the waves propagated along IL′ and I′L″ may be either in complete accordance or discordance, this space must be a multiple of $\frac{l}{2}$, or $(p - p')$ must be some number of the natural series.

To find an expression for $p - p'$, denoting I″Q the perpendicular from I″ to II′ by e, and the angles of incidence and refraction by i and r respectively, we have

$$p' = \frac{\mathrm{IP}}{\frac{1}{2}l'} = \frac{\mathrm{II'} . \sin i}{\frac{1}{2}l'} = \frac{2\mathrm{IQ} . \sin i}{\frac{1}{2}l'} = \frac{2e . \sin r . \sin i}{\frac{1}{2}l' . \cos r},$$

and $$p = \frac{\mathrm{II''I'}}{\frac{1}{2}l} = \frac{2\mathrm{II''}}{\frac{1}{2}l} = \frac{2e}{\frac{1}{2}l . \cos r}.$$

$\frac{I'I'}{II'} = \sin r$

$\frac{II'}{II''} = \cos r$

Hence $$p - p' = \frac{2e}{\cos r}\left(\frac{1}{\frac{1}{2}l} - \frac{\sin i}{\frac{1}{2}l'} . \sin r\right)$$

$$= \frac{4e}{\cos r}\left(\frac{1}{l} - \frac{\sin i}{l'} . \sin r\right).$$

But $$\frac{\sin i}{\sin r} = \frac{l'}{l}, \text{ or } \frac{\sin i}{l'} = \frac{\sin r}{l};$$

and hence $$p - p' = \frac{4e}{\cos r}\left(\frac{1}{l} - \frac{\sin^2 r}{l}\right)$$

$$= \frac{4e}{\cos r}\left(\frac{1 - \sin^2 r}{l}\right)$$

$$= \frac{4e \cos^2 r}{l \cos r} = \frac{4e \cos r}{l}.$$

Denoting then by p''' any number of the natural series, we have

$$p''' = \frac{4e \cos r}{l};$$

and hence $$l = \tfrac{1}{4}p''' . l . \frac{1}{\cos r} = \tfrac{1}{4}p''' \, l \sec r.$$

From this equation it appears,

1°. That the thicknesses at which the successive rings are formed, are as the numbers of the natural series.

2°. That for different kinds of light, these thicknesses are as the lengths of the waves.

3°. That for different obliquities, they are as the secants of the angles of refraction, or of the angles of incidence on the exterior medium.

4°. That for plates of different substances, they are directly as the lengths of the waves, and hence inversely as the refractive indices of the substances of which the plates are composed. Thus, according to the theory, the rings formed by oblique light observe the same laws as those formed at a perpendicular incidence.

145. *Explanation of the transmitted rings.*

The formation of the second system of rings is accounted for by the interference of the rays which are transmitted directly, with those which are transmitted after two reflections within the plate.

Thus LI [Fig. 111] being an incident ray, the portion of it which is transmitted consists of two parts, one of which passes directly through, while the other is first reflected from the second surface of the plate at I′ to I, and from the first surface at I back again; so that, denoting the thickness as before by e, if

$$2e = \frac{l}{2},$$

or any odd multiple of $\frac{l}{2}$, the undulations will be in complete discordance; but if

$$2e = 2\frac{l}{2},$$

or any even multiple of $\frac{l}{2}$, they will be in complete accordance.

For although the direction of the vibratory motion in the latter system of undulations is changed at I′, it is restored at I; and consequently both

systems pass through I′, having their vibrations in the same direction.

Thus the thicknesses corresponding to the dark transmitted rings will be as the numbers

1, 3, 5, 7, 9, etc.;

those corresponding to the bright transmitted rings, as the numbers

0, 2, 4, 6, 8, etc.

145′. To account for the fact that the color reflected at any thickness of the plate is complementary to that which is transmitted at the same thickness, let the mass of the ether in each of the three systems of undulations, the incident, reflected and transmitted, be denoted by m; and let the velocities of vibration in these systems be denoted by v, v' and v'' respectively: then by the law of the conservation of living forces, we have

why is m the same in medium?

$$mv^2 = mv'^2 + mv''^2,$$

and hence $$v^2 = v'^2 + v''^2;$$

that is, since v^2, v'^2 and v''^2 measure the intensities of the light in the respective systems, the sum of the intensities of the reflected and transmitted undulations is equal to the intensity of the incident undulations; or, in other words, the intensities of the reflected and transmitted rays are complementary to each other. When, therefore, a ray of compound or white light is incident at any point of the plate, the intensity of each of the simple

rays in the reflected light will be complementary to the intensity of the ray of the same color in the transmitted light; and hence the color resulting from the combination of all the simple rays reflected at that point, will be complementary to that produced by the union of the simple rays transmitted at the same point. If, for example, twice the thickness of the plate, at the point of incidence, is just equal to an odd multiple of $\frac{l}{2}$ for red light, then in the reflected undulations there will be a maximum of red light, a little less than a maximum of orange, still less of yellow, and so on; while in the transmitted undulations there will be a minimum of red light, a little more than a minimum of orange, still more of yellow, and so on; the whole resulting in the production of compound reflected and transmitted rays of complementary tints.

CHAPTER VIII.

DIFFRACTION.

146. WHEN a pencil of homogeneous light, diverging from S [Fig. 114] an exceedingly minute origin, is in part intercepted by an opaque body *a*G, and the shadow of the body is received on a screen EE′ of white paper or ground glass, the following phenomena are observed:

1°. The light, instead of ceasing abruptly at P the edge of the geometric shadow, extends to a small but sensible distance within it, gradually and continuously diminishing as it recedes from P.

2°. The part of the screen immediately on the right of P is crossed by a series of bands or fringes, alternately bright and dark, similar to those formed in the experiment of the mirrors [Art. 126]; the light gradually increasing and diminishing to lines of maximum and minimum intensity, for a number of alternations.* These bands are parallel to the edge of the shadow (supposed in the figure to be perpendicular to the plane of the paper); and their distances from it, and from each other, vary with

* There is never, however, a total absence of light, as in the experiment of the mirrors, even in the lines of minimum intensity.

the distance of the screen from the body or obstacle, diminishing indefinitely as the screen is moved towards the latter.

3°. When the points of maximum or minimum intensity of a fringe of the same order, bright or dark, are determined at different distances from the obstacle, they are found to lie on a hyperbola, of which the foci are at S the luminous point, and *a* the edge of the obstacle.

4°. When the luminous point is made to approach the edge of the obstacle, the positions of the screen and the obstacle remaining the same, the distance between a fringe of a given order and the edge of the geometric shadow is found to increase.

5°. The breadth of the fringes varies with the color of the light; being least in violet light, and increasing in the order of the prismatic spectrum to the red.

147. If the experiment be varied by employing a very narrow obstacle, a fine wire, for example, then not only will fringes be formed on both sides of the geometric shadow of the wire, analogous to those in the preceding experiment, but also in the space occupied by the shadow itself: those without the shadow, are called exterior fringes; those within, interior fringes.

If the breadth of the obstacle be increased, a limit is soon reached, at which the interior fringes disappear, and the case becomes entirely similar to that of the preceding article.

The interior fringes immediately disappear, when the light that passes by either side of the wire is intercepted.

148. If the experiment be again varied by employing an obstacle pierced with a very narrow rectangular aperture, and permitting no light to pass except through it, then it will be found that the illuminated space on the screen is much broader than the geometrical projection of the aperture; and that a greater or less portion of it, according to the distance of the screen from the obstacle, is occupied by fringes, symmetrically distributed on each side of the line which bisects the projection lengthwise. Thus, at a certain distance of the screen, only exterior fringes are seen, or fringes without the projection of the aperture: if the distance be somewhat diminished, interior fringes, or fringes within the projection, become visible; and, at a still less distance, the exterior fringes vanish, the interior remaining, but undergoing very sensible changes as the distance is made to vary.

In both the above cases, the fringes are propagated from the obstacle in hyperbolas, and their breadth varies with the color of the light.

As yet we have supposed the light to be homogeneous. When the light employed is white, iris-like fringes take the place of the alternate bright and dark bands; an obvious consequence of the variation in breadth of the simple bands, with the color of the light by which they are formed.

149. The preceding experiments may be indefinitely varied, producing effects more or less complicated. The phenomena, taken together, constitute a branch of optical science, called the *diffraction of light.* They have been shown by YOUNG and FRESNEL to be necessary consequences of the wave theory. The explanation of them is based upon the following assumption or principle, relative to the mode of action of the ether, known as *the principle of Huygens.*

Let OO′ [Fig. 115] be the surface of a spherical wave, originating at a luminous point S. Then each point of this surface may be regarded as a new centre of disturbance, from which vibrations are propagated in all directions in front of the wave. Hence to determine what will be the motion of a point P in front of OO′, when the wave shall have reached it, we must consider all the points of OO′ as distinct centres of agitation, from which motion is transmitted to P.

At any point of OO′, as *w*, for example, the motion transmitted in the primitive direction *wp* normal to the surface, must be supposed to be greater than that transmitted in any oblique direction, as *wq;* and the phenomena of light indicate that the intensity diminishes rapidly as the obliquity, or the angle *pwq*, increases; so that this angle soon attains a magnitude at which the motion transmitted in the line *wq* is insensible.

Now from P let the straight lines Pb, Pc, Pd, etc. be drawn, intersecting OO′ at the points b, c, d, etc. so taken that any one of the lines shall exceed that which immediately precedes it by half an undulation. We shall then have

$$Pb - Pa = Pc - Pb = Pd - Pc = \text{etc.} = \tfrac{l}{2}.$$

It is evident that the arcs ab, bc, cd, etc. diminish as they recede from a; and that at first the diminution is very rapid, so that bc is much less than ab, cd than bc, etc. But this rapid variation in magnitude soon ceases, and, at a little distance from a, the difference between any two consecutive arcs becomes so small that it may be neglected; for since $\frac{l}{2}$ is almost infinitely small, compared with the distance from P to any point of OO′, any two consecutive lines which intersect that arc at a little distance from a, as Pr and Ps, for example, may be regarded as parallel, and hence the angles Pro, Pso which they make with the arc as equal: consequently the triangles ovr, rus formed by describing from P as a centre the arcs ov, ru, may be considered equal, and hence also the arcs or and rs.

Let us now consider the action of any two consecutive arcs, as bc, cd, upon the molecule at P, the arcs being supposed to be so near a as to differ in magnitude. On the arc bc, and very near b, let the point x be taken; and on cd, let y be taken so that yP — xP shall be equal to $\frac{l}{2}$. Then the vibrations

from the arcs bx, cy will reach P in exactly opposite phases; and were the arcs equal, and the directions Pb, Pc parallel, the effect on P would be zero; but as bx is greater than cy, and Pb less oblique than Pc, the motion transmitted from bx to P will exceed that transmitted from cy. The same is evidently true of all the corresponding portions of bc, cd. Hence, 1st, the vibrations transmitted to P by the whole arcs bc, cd, will be in complete discordance; and 2d, the motion transmitted by the first arc will exceed that transmitted by the second; and generally the vibrations transmitted to P by any one of the arcs ab, bc, cd, etc., will be in complete discordance with the vibrations transmitted to the same point by the adjacent arc to the right; and of the motions transmitted to P by the two arcs, that transmitted by the first will be the greater.

Denote the motions transmitted to P by the arcs ab, bc, cd, etc. by m, m', m'', etc. respectively; and their sum, or the motion transmitted by the indefinite arc aO, by s; we shall then have the equation

$$s = m - m' + m'' - m''' + m^{\text{iv}} - m^{\text{v}} + \text{etc.}$$

In the series composing the second member of this equation, each term is, as has just been shown, greater than any one of the succeeding terms; but as the arcs at a little distance from a may be considered equal, and their directions from P parallel, the terms of the series will rapidly approach equality,

so that the value of s will depend upon $m - m'$ and a few of the terms immediately following.

We have confined our attention to the arc aO; but whatever is true of it, is equally true of the arc aO′: besides, the motions which the two arcs transmit to P are evidently equal to each other. We therefore conclude that the only part of the wave OO′ which actually produces motion at P, is a small arc immediately contiguous to a, and bisected at that point. In reference to the point P, this arc may be called the effective portion of OO′.

If we suppose the light which emanates from S to be received on a screen, all the points of which are at the same distance from S as the point P, the illumination of the screen will evidently be uniform, the intensity at each point being that due to the motion $2s$. Let us see how this intensity should be modified, when a portion of the light is intercepted.

We shall consider the three cases in which the experimental results have already been given. And, first, let the part of the wave which lies to the left of a [Fig. 116] be intercepted by an opaque body. Then, at P the edge of the geometric shadow, the intensity is evidently that which is due to s the motion transmitted by aO. At any point, as P′, a little within the shadow, there must also be motion due to the same arc aO; but the intensity at this point is less than at P, since the wave acts more obliquely upon P′ than upon P; and at some point P″ at which the direction is so oblique that the

action of the wave upon it is insensible, the intensity is zero, and the *physical shadow* begins. Thus, according to the theory, the intensity, in proceeding from P to the left, should decrease gradually, and not be reduced to zero till at a small but sensible distance from the edge of the geometric shadow.

At a point P‴ on the right of P, such that

$$P'''a + aS - P'''S = \tfrac{l}{2},$$

the intensity is that due to s the motion transmitted by the indefinite arc a'O, plus m the motion transmitted by the arc aa'; but as

$$-m' + m'' - m''' + m^{\text{IV}} \ldots . \text{ etc.}$$

is negative, and hence m greater than s, this intensity is actually greater than it would be were no light intercepted. or greater than 2s.

At P^{IV}, for which

$$P^{\text{IV}}a + aS - P^{\text{IV}}S = 2\tfrac{l}{2},$$

the intensity is due to the motion $s + m - m'$, a quantity greater than s, but less than $2s$, and is therefore less than if no light were intercepted, but greater than the intensity at P.

At P^{V}, for which

$$P^{\text{V}}a + aS - P^{\text{V}}S = 3\tfrac{l}{2},$$

the intensity is due to $s + m - m' + m''$, or $s + m - (m' - m'')$, a quantity greater than $2s$, but less than $s + m$. At this point, therefore, the intensity is again greater than if no light were intercepted, though less than the intensity at P‴.

At P^{VI}, for which

$$P^{VI}a + aS - P^{VI}S = 4\frac{l}{2},$$

the intensity is due to $s + m - m' + m'' - m'''$, a quantity less than $2s$, but greater than $s + m - m'$. Hence, at this point, the intensity is again less than if no light were intercepted, but greater than the intensity at P^{IV}.

Thus on the right of P, a series of points may be found, at which the intensities are alternately greater and less than when no light is intercepted; the maximum intensities constantly diminishing, and the minimum increasing, till they reach a common limit in the intensity due to $2s$. This occurs when we arrive at a point on the screen, so remote from P, that the motion transmitted to it from a may, on account of the obliquity of the direction, be neglected. The formation of the fringes, then, is satisfactorily explained.

150. It remains to account for the phenomena 3°, 4°, and 5° of Art. 146.

To deduce from the theory the curvilinear propagation of the fringes, let Z [Fig. 116] be a point of maximum or minimum intensity, for which we have

$$Za - Zw = n\frac{l}{2};$$

and put $Sa = Sw = r$, $aP = x$, and $PZ = y$.

We then have

$$SZ = \sqrt{(r + x)^2 + y^2}, \text{ and } Za = \sqrt{x^2 + y^2}.$$

But $$Zw = SZ - Sw;$$

and hence, by substitution, we get

$$Zw = \sqrt{(r+x)^2 + y^2} - r.$$

Substituting these values of Za and Zw in the equation $Za - Zw = n\frac{l}{2}$, we find

$$\sqrt{x^2+y^2} - \sqrt{(r+x)^2+y^2} + r = n\tfrac{l}{2};$$

and hence

$$\sqrt{(r+x)^2+y^2} - \sqrt{x^2+y^2} = r - n\tfrac{l}{2}.$$

Now this is evidently the equation of an hyperbola, which has its foci at the points a and S, and of which $(r - n\frac{l}{2})$ is the transverse axis, and x and y the co-ordinates. Hence as the screen EE′ is moved towards or from a the edge of the obstacle, while the distance between the luminous point and the obstacle remains the same, the point Z describes the arc of an hyperbola.

Resuming the equation

$$\sqrt{x^2+y^2} - \sqrt{(r+x)^2+y^2} + r = n\tfrac{l}{2},$$

we get by transposition

$$\sqrt{(r+x)^2+y^2} = \sqrt{x^2+y^2} + (r - n\tfrac{l}{2}).$$

Squaring each member of this equation, and reducing, we find

$$\sqrt{x^2+y^2} = \frac{2rx + nlr - n^2\frac{l^2}{4}}{2r - nl};$$

and, hence, executing the division indicated in the second member

$$\sqrt{x^2+y^2} = x + \frac{nl}{2} + \frac{nlx + \frac{1}{4}n^2l^2}{2r - nl}.$$

a From this equation, it appears, that if we suppose x to remain invariable, PZ. y increases Sa-SW. as r diminishes; that is, if, while the distance between the obstacle and the screen remains the same, we cause the luminous point to approach the obstacle, the distance between a given fringe and the edge of the geometrical shadow will increase.

The cause of the variation in the breadth of the fringes, as the color of the light is changed, is sufficiently obvious.

151. Let us now suppose the obstacle reduced to an exceedingly narrow rectangular plane, of which aa' [Fig. 117] is a section perpendicular to its length, and PP′ its geometrical shadow; and p' being any point in the shadow, let the points b, c, d, etc., b', c', d', etc., be so taken that

$$p'b - p'a = p'c - p'b = \ldots . = \tfrac{l}{2},$$

and
$$p'b' - p'a' = p'c' - p'b' = \ldots . = \tfrac{l}{2}.$$

The motion of the molecule at p' is due to the motions transmitted by the arcs aO, a'O′.

The motion transmitted to p' by the arc aO is the resultant of the motions transmitted by the arcs ab, bc, cd, etc. The direction of this resultant is some line $p'x$ to the right of $p'a$, and making with $p'a$ a very small angle. This angle is not invariable, and a little consideration makes it apparent that it increases as p'P diminishes, Sz remaining the same; and diminishes as Sz diminishes, p'P remaining the same.

All that we have said of the arc aO and its resultant, is equally true of a'O$'$ and its resultant. Let the direction of the resultant of the latter arc be $p'x'$; then according as $p'x' - p'x$ is an odd or even multiple of $\frac{l}{2}$, the intensity at p' is a minimum or maximum. The intensity at p the middle of PP$'$ is evidently always a maximum.

There will thus be a series of points of maximum and minimum intensity within the geometric shadow, symmetrically distributed on PP$'$ to the right and left of p. In this case, therefore, there will be a system of fringes within the shadow.

If aa' be equal to that part of a'O$'$ which would be effective at P$'$ if not obstructed, the fringes on the left of P$'$ will be identical with those in the preceding case. For any less value of aa', they will obviously be more or less modified by the light transmitted from aO.

From the foregoing explanation, it appears that if a very small circular disc were substituted for the narrow rectangular plane, the space occupied by the shadow should present the appearance of a brilliant spot at its centre, surrounded by rings alternately dark and bright. This deduction from the theory has been verified by experiment.

152. We will now consider the case in which the light that falls upon the screen is supposed to pass through a very narrow rectangular aperture.

Let aa' [Fig. 118] be a transverse section of the aperture; and let the screen EE$'$ be at such a dis-

tance, that for any point p' of the projection PP′ of the aperture, we shall have $p'a' - p's'$, $p'a - p's'$ each less than a semi-undulation: then it is evident that there will be no points of maximum and minimum intensity on PP′, and consequently no interior fringes. The same is true of all positions of the screen more distant from the aperture than EE′.

But in this position of the screen, there may be exterior fringes. To show this, let a point be supposed to move on EE′, from a coincidence with P′ towards the left; the difference of its distances from a and a', the edges of the aperture, will evidently become successively equal to 1, 2, 3, etc. semi-undulations. If p'' be a position of this point, for which we have

$$p''a - p''a' = n \tfrac{l}{2},$$

we may suppose aa' to be divided into n parts, such that the difference of the distances from p'' to any two consecutive points of division shall be equal to a semi-undulation. Any two consecutive parts will then transmit to p'' undulations completely discordant, and hence will nearly, if not entirely neutralize each other. Consequently if n be an even number, all the parts of the arc will be neutralized, or nearly so, and the intensity at p'' will be a minimum; but if n be odd, one part will remain uncompensated, and the intensity will be a maximum.

We may thus have fringes without the projection of the aperture, when, on account of the distance of the screen, none are formed within it.

If now we suppose the screen to be moved towards the aperture, the difference of the distances from p the middle of PP′, to s the middle of the aperture, and from p to a or a' one of its edges, will become successively equal to 1, 2, 3, etc. semi-undulations. Let E″E‴ be a position at which we have $pa - ps$, $pa' - ps$ each equal to $n \cdot \frac{l}{2}$; then, according as n is odd or even, the intensity of the light at p will evidently be a maximum or minimum; so that as the screen approaches the aperture, p will be alternately bright and obscure.

There will also be other points of maximum and minimum intensity on P″P‴. For on the right of p there will be a point p''', for which we shall have (very nearly)

$$p'''a' - p'''s'' = (n + 1)\frac{l}{2}, \text{ and } p'''a - p'''s'' = (n - 1)\frac{l}{2};$$

and at which the intensity will be a maximum or minimum, according as n is even or odd. Still further to the right there will be a point p^{IV} for which we shall have (very nearly)

$$p^{\text{IV}}a' - p^{\text{IV}}s''' = (n + 2)\frac{l}{2}, \text{ and } p^{\text{IV}}a - p^{\text{IV}}s''' = (n - 2)\frac{l}{2};$$

and at which the intensity will be a maximum or minimum, according as n is odd or even, and so on. There will be a similar series of points on the left of p.

If the screen be supposed to approach still nearer to the aperture, a position $E^{iv}E^{v}$ will at length be reached, at which the lines from a to P^{v} and from a' to P^{iv} are so oblique that the motions transmitted in these directions are insensible. The external fringes will then vanish, and the interior fringes will exist alone.

153. When the light is transmitted through two adjacent rectangular apertures of the same size and in the same plane, as represented in figure 119, fringes are observed on the right of P and on the left of Q; and if the screen is not too distant, also between P and P′, and Q and Q′. But that which especially characterizes this case, is a system of very narrow and vivid fringes between P′ and Q′, the central fringe being at p the middle point of P′Q′. When the apertures are very narrow, and the screen so distant that the difference of the distances from any point of P′Q′ to the points a and a' is much less than the length of a semi-undulation, the undulations proceeding from aa' may be supposed to arrive at P′Q′ in directions sensibly parallel, and in the same phase of vibration; and, as the same may be supposed true of those proceeding from bb', the experiment, in its essential circumstances, becomes entirely similar to that of the slightly inclined mirrors [Art. 126]. In verification of this, the lengths of the undulations, as determined by the two methods, are found to be identical.

154. Since light moves with the less velocity in the more refractive medium [Art. 137], it is evident that if in the experiment of the preceding article a very thin plate of any transparent substance be interposed in the light propagated from aa', the central fringe will be shifted to some point p' on the right of p, such that $p'a$ and $p'b$ shall contain the same number of semi-undulations, and hence the light be transmitted from aa', bb' to p' in the same time; and that according as the thickness of the plate contains one, two, three, etc. undulations more than the thickness of an equal plate of air, the stripe at p' will be the first, second, third, etc. bright stripe in order from p'.

Let the thickness of the plate be denoted by e; the index of refraction for light passing from air into the substance of the plate, by n; the lengths of the undulations in air, and in the plate, by l and l' respectively; the number of times l and l' are contained in the thickness e, by m and m' respectively; and the order of the fringe at p, by x.

We shall then have

$$e = ml = m'l', \text{ and hence } \frac{l}{l'} = \frac{m'}{m};$$

$$m' - m = x, \quad \text{and [Art. 137] } n = \frac{l}{l'};$$

and consequently

$$n = \frac{m'}{m} = \frac{m+x}{m} = 1 + \frac{x}{m}$$

$$= 1 + \frac{xl}{e}.$$

The value of l for light of each of the primary colors is known, and that of x can be determined with great accuracy; by means of this equation, therefore, we can find the value of n when e is given, and conversely that of e when n is given. This method of determining the value of n is susceptible of extreme accuracy, and can be employed in cases in which the minuteness of the quantities to be ascertained would render the ordinary methods inapplicable.

155. We have thus deduced, from the principle of interferences, an explanation of the fundamental cases of diffraction. The same fertile principle has been successfully employed to explain other analogous phenomena, as the colors produced by fine gratings, striated surfaces, etc. The student who wishes to pursue the subject, is referred to LLOYD'S Lectures on the Wave Theory; and to the Treatises on Physics, of POUILLET and DAGUIN.

PART SECOND.

POLARIZED LIGHT.

CHAPTER I.

DOUBLE REFRACTION.

156. When a pencil of light falls upon the surface of a transparent medium, the refracted portion forms either a single pencil, or two distinct pencils, according to the nature of the medium. The first case has already been considered: the latter now claims our attention.

Substances which possess the property of refracting light into two separate pencils, are called *doubly refracting substances*, and the phenomenon itself is called *double refraction.*

The class of doubly refracting substances comprehends all crystallized media, as transparent salts, gems, and crystallized minerals, of which the primitive form is not a regular polyhedron; and all animal and vegetable substances in which there is any tendency to a crystalline arrangement of the particles, as hair, horn, shells, certain leaves, seeds, etc.

The property of refracting light doubly, may be imparted to certain artificial substances which do not possess it in their ordinary state, by subjecting them unequally to pressure, or change of temperature. The principal substances of this class are glass, gums, resins, and jellies. Liquids, gases and vapors, in general, refract light singly under all circumstances.*

The angle by which the two pencils are separated varies with the nature of the substance, and the direction in which the light is incident. In most substances this angle is so large as to be easily observed and measured, and in some bodies it is very considerable. In some substances, in which it is so small as to escape observation, the separation of the light is inferred from certain phenomena which are known to depend upon the existence of two refracted pencils.

In every crystal which exhibits double refraction, there is at least one direction in which a pencil of light may be transmitted without being divided. In the greater number of crystals, there are two such directions. These directions are called the *axes of double refraction*, or *the optical axes* of the crystal.

Those crystals in which there is but one direction of indivisibility, are called *crystals of one axis;* those in which there are two such directions, are called *crystals of two axes.*

* Certain exceptions will be noticed hereafter.

157. CRYSTALS OF ONE AXIS.

Double refraction was first observed in carbonate of lime. This mineral is a crystal of one axis, and, producing a very decided separation of the pencils, is usually selected to illustrate the laws of the phenomena in the class to which it belongs. Its primitive form is that of a rhombohedron; that is, a crystal of carbonate of lime, whatever its form, may be supposed to be made up of a great number of exceedingly minute rhombohedral crystals, so disposed as to have their corresponding faces parallel to each other. If the vertices of the obtuse angles of these ultimate crystals or molecules be conceived to be joined by straight lines, we shall have within the crystal a system of parallel lines or axes, the position of which, with respect to the edges of the molecules, is fixed, and is not affected by any change that may be made in the form of the crystal. If the crystal be made to assume the form of an exact rhombohedron, by cutting it in directions parallel to its natural faces, then by joining the vertices of its obtuse angles by a straight line, we shall have the common direction of the axes of its molecules. This line is evidently the *axis of form* of the crystal; that is, the matter of the crystal is symmetrically disposed about it.

In crystals of many other substances, and of other primitive forms, the matter is in like manner symmetrically distributed about a single line or axis.

It has been found that all crystals, which have an axis of form, have but one optical axis, and that these two axes are parallel to each other; the optical axis being not a single line passing through a definite point, but, as from the structure of the crystal it obviously should be, simply a direction. Thus, if we cut a crystal [Fig. 120] of the class in question, a crystal of carbonate of lime for example, in the direction of two planes perpendicular to its axis of form, and remove the solid angles, it will be found that an incident pencil, perpendicular to either of the artificial faces produced, is not divided. If we remove but one of the solid angles, and cause the pencil to fall perpendicular to the remaining artificial face, still there will be no double refraction; and the pencil, as it emerges from the second surface, will be refracted according to the ordinary law. Hence if the emergent pencil should become the incident pencil, we should expect the light to retrace its course, passing through the crystal in the direction of the axis, and entering the air perpendicular to the surface of emergence. This is verified by experiment. Thus, whatever the obliquity of the incident pencil, and whether it enter by a natural or artificial face, it never suffers double refraction when it passes through the crystal in the direction of its axis.

Whenever the light does not traverse the crystal in the direction of its axis, it is divided into two pencils more or less inclined to each other, which, if the surfaces of incidence and emergence are parallel, emerge parallel to the incident pencil. One of these always obeys both the fundamental laws of refraction; the other, except in certain cases, obeys neither of them. The first is called *the ordinary pencil* ; the latter, *the extraordinary pencil.* Thus, while the ordinary ray remains in the plane of incidence, the extraordinary ray in general does not; and if we denote the angle of incidence by i, the angles of refraction of the ordinary and extraordinary rays by r and r', and the corresponding indices by n_o and n_e, then in the equations

$$\frac{\sin i}{\sin r} = n_o, \qquad \frac{\sin i}{\sin r'} = n_e,$$

while n_o is constant, n_e is in general variable.

158. In several positions of the face and plane of incidence relative to the axis, the refraction of the extraordinary ray is characterized by certain peculiarities, which we proceed to make known.

A plane parallel to the axis, and perpendicular to any face of the crystal, natural or artificial, is called *a plane of principal section.* When the plane of incidence coincides with this plane, the extraordinary ray observes the first law of ordinary refraction, and both refracted rays lie in the same plane.

To give a general idea of the refractions as they take place in this plane, let ABA′B′ [Fig. 121] be a principal section of one of the natural faces of the crystal, and let the light be incident in the plane of the section; then when the incident ray coincides with the normal LO, of the two portions into which it is divided, the one, the ordinary ray, also coincides with LO; while the other, the extraordinary ray, is refracted in some direction IE, so as to make with the axis drawn through the point of incidence, a greater angle than the ordinary ray.

When the incident ray L′I or L″I is inclined to the normal, both the transmitted rays suffer refraction; the extraordinary ray IE′ or IE″, still making with the axis a greater angle than the ordinary ray IO′ or IO″. The exact position of the extraordinary ray is determined by a complex law.

159. Let us now suppose the form of the crystal (which we will still suppose to be of carbonate of lime) to be changed, by grinding and polishing, into that of a rectangular parallelopipedon AA′BB′ [Fig. 122] having its lateral faces parallel to the axis XX′, and its bases perpendicular to it. And first let the light be supposed to fall upon one of the bases; then a ray perpendicular to the face, and hence coinciding with the axis drawn through the point of incidence, is, as we have already seen, not divided; but an oblique ray, as LI, suffers double refraction; and the extraordinary ray, remaining in the plane of incidence, but not obeying the law of the

sines, makes, as in the preceding case, a greater angle with the axis than the ordinary ray. If the incident ray be made to revolve about the point I, without changing its inclination to the face, the inclination of the extraordinary ray, as well as that of the ordinary ray, will remain constantly the same; and thus each ray will generate the surface of a right cone, having for its axis the optic axis at I. This constancy of the angle of extraordinary refraction for the same angle of incidence, whatever the plane of incidence, is peculiar to the faces perpendicular to the axis.

160. In the case in which the light is incident upon one of the lateral faces of the parallelopipedon, there are two positions of the plane of incidence worthy of especial consideration: the one, that in which the plane is perpendicular to the axis; the other, in which it coincides with the principal section of the surface.

1°. When the plane of incidence is perpendicular to the axis, it is found that both the transmitted rays are subjected to to the ordinary laws of refraction; that is, that both rays remain in the plane of incidence, and that n_e the ratio of extraordinary refraction is constant. The constant value which n_e assumes in this position of the surface and plane of incidence, is also found to be its minimum value; and to distinguish it from other values of this index, we enclose it in brackets thus (n_e).

The numerical values of n_o and (n_e), in the case of carbonate of lime, are 1.65 and 1.48 respectively, (n_e) being less than n_o. But (n_e) is not less than n_o in all crystals of one axis: in some it is found to be greater; and hence we have a further division of crystals of this class, into, 1st, crystals in which (n_e) is less than n_o, called *negative crystals*; and 2d, répulsive crystals in which (n_e) is greater than n_o, called *positive crystals*. attractive

If the reciprocals of n_o and (n_e) be denoted by b and a respectively, we have [Art. 157],

$$\frac{\sin r}{\sin i} = \frac{1}{n_o} = b, \quad \text{and } \frac{\sin r'}{\sin i} = \frac{1}{(n_e)} = a.$$

The positions of the refracted rays may, in the present case, be determined by the following simple geometrical construction:

Let the plane of incidence be supposed to coincide with the plane of the paper, and let YY′ [Fig. 123] be the intersection of this plane with the face of the crystal on which we suppose the light to fall; then, on that part of the plane of incidence which lies below this face, and from I the point of incidence as a centre, and with IB equal to b and IK equal to a as radii, let the circles ABC, HKR be described. Let LI be the incident ray, and ID a line perpendicular to it. Also let FD be a line equal to unity, inscribed in the angle DIF, and perpendicular to ID. Then if from the point F where FD meets the crystal, we draw FO and FE tangent

to these circles, the straight lines IO, IE will be the directions of the ordinary and extraordinary rays respectively.

For the angles LIP and FID are evidently equal, as are also the angles OIN and OFI, EIN′ and EFI. Hence, denoting as before the angles LIP, OIN and EIN′ by i, r and r' respectively, we have

$$\sin i = \sin \text{FID} = \frac{\text{FD}}{\text{IF}} = \frac{1}{\text{IF}}.$$

But $$\frac{\text{IO}}{\text{IF}} = \sin \text{OFI} = \sin \text{OIN} = \sin r;$$

and hence $$\frac{1}{\text{IF}} = \frac{\sin r}{\text{IO}} = \frac{\sin r}{\frac{1}{n_o}} = n_o \,.\, \sin r;$$

consequently

$$\sin i = n_o \,.\, \sin r.$$

Again we have

$$\frac{\text{IE}}{\text{IF}} = \sin \text{EFI} = \sin \text{EIN}' = \sin r';$$

and hence

$$\frac{1}{\text{IF}} = \frac{\sin r'}{\text{IE}} = \frac{\sin r'}{\frac{1}{(n_e)}} = (n_e) \,.\, \sin r',$$

and consequently

$$\sin i = (n_e) \sin r'.$$

2°. When the plane of incidence coincides with the principal section of one of the lateral faces of the prism, for the ordinary ray we have as usual

$$\frac{\sin i}{\sin r} = n_o;$$

while for the extraordinary ray (which also remains in the plane of incidence), the following relation between the tangents of the two angles of refraction, and the constants a and b, is found to exist:

$$\text{tang } r' = \frac{b}{a} \cdot \text{tang } r.$$

The method of determining the direction of the ordinary ray by a geometrical construction, is the same as in the foregoing case; thus supposing I [Fig. 124] to be the point of incidence, and the plane of incidence to coincide with the plane of the paper, we determine as before the point O, and draw the straight line IO: it will be the direction required. To find the direction of the extraordinary ray, from I as a centre, describe an ellipse AKA′, having its semi-conjugate axis IA equal to b, and its semi-transverse axis IK equal to a, the conjugate coinciding with XX′ the direction of the axis of the crystal; then, from the point F draw FE tangent to the ellipse: IE will be the position of the ray.

For

$$\text{tang } r = \text{tang OIN} = \text{tang IOQ} = \frac{\text{IQ}}{\text{QO}},$$

and

$$\text{tang } r' = \text{tang EIN}' = \text{tang IEQ} = \frac{\text{IQ}}{\text{QE}};$$

and hence

$$\text{tang } r \ : \ \text{tang } r' \ :: \ \text{QE} \ : \ \text{QO}.$$

But by a property of the ellipse, we have

$$QE : QO :: IK : IB$$
$$:: a : b;$$

consequently

$$\text{tang } r : \text{tang } r' :: a : b,$$

and
$$\text{tang } r' = \frac{b}{a} \text{tang } r.$$

161. The law which governs the direction of the extraordinary ray in the general case (in which neither of the laws of ordinary refraction is observed), is very complex; but in this, as in the foregoing cases, its position can be determined geometrically.

Thus, let MM′ [Fig. 125] be any face of the crystal, natural or artificial; LI any incident ray, and RIR′ the trace of the plane of incidence. Through I draw a straight line AIA′ parallel to the axis of the crystal (this will not in general be in the plane of MM′), and on it lay off IA, IA′ each equal to b. On AA′ as an axis, describe an ellipsoid of revolution, having the points A, A′ as poles, and the radius of its equator equal to a. On IR′, determine the point F in the same manner as in the first case, and through it draw TT′ perpendicular to the plane of incidence. Lastly, through TT′ draw a plane tangent to the ellipsoid: the straight line drawn from I to E the point of contact, will be the position of the extraordinary ray.

If, instead of the ellipsoid, a sphere be described on AA′ as a diameter, and through TT′ a plane be drawn tangent to it, the line joining I and the point of contact O will be the position of the ordinary ray.

To verify this construction, the principles involved in it are assumed to be true, and general formulæ investigated for determining the positions of the rays. It is found that in all cases the deductions from these formulæ agree with the results of observation; that is, if we give to the crystal, by grinding and polishing, faces inclined at any angle to the axis, and determine with exactness the positions of the refracted rays for any incident pencil, and then referring to the general formulæ, and adapting them to the particular positions of the face and plane and angle of incidence in the case in question, deduce from them the positions of the rays, the two determinations, the hypothetical and experimental, are found to accord.

The above elegant constructions were given by HUYGENS: they are not empirical, but are necessary consequences of the theory of undulations.

162. *Crystals of two axes.*

In crystals of two axes, the phenomena are more complex than those we have been considering; in general, neither of the rays being refracted according to the ordinary laws.

CHAPTER II.

PHENOMENA AND LAWS OF POLARIZATION.

163. When either of the two portions into which a pencil of light is divided by transmission through a doubly refracting medium, is subjected to certain tests, it is found to differ essentially from a beam received directly from the sun, or other self-luminous source. In its passage through the medium, the light has acquired peculiar properties. The same properties may be imparted by reflection, or by ordinary refraction. Light which possesses these properties, is said to be polarized. The nature of the properties of polarized light will be developed as we proceed.

If a beam of light LI [Fig. 126], emanating directly from the sun, be received upon a plate of glass MN, and the plate be made to revolve about the beam as an axis, in such a manner that the angle of incidence, having any value whatever, may not vary during the revolution, the intensity of II′ the reflected portion of the incident light will be the same in every position assumed by the plate, or by the plane of reflection LII′. But if such a position be given to MN, that the light may fall upon it at an angle of 54° 35′ of incidence,

and the reflected pencil II′ be received upon a second plate of glass M′N′ at the same angle, and, while MN remains fixed, the plate M′N′ be made to revolve about II′ at a constant angle of inclination, the intensity of the second reflected pencil I′L′ will be found to vary as the plate revolves.

Let the variable intensity of this pencil be denoted by i, and its maximum value by a; then when the position of M′N′ is such that the planes of first and second reflection of LII′, II′L′ coincide, i is found to be equal to a. As the plate revolves, i diminishes; and when the position M″N″ [Fig. 127] is attained, in which the planes of first and second reflection are at right angles to each other, it is reduced almost to zero, nearly all the light passing through the plate. The plate continuing to revolve, i increases, and, when the planes of reflection again coincide, is as before equal to a. Through the remaining 180°, the same appearances are presented; the intensity of I′L′ having its maximum value when the planes of reflection coincide, and being reduced almost to zero when they are at right angles to each other, and passing from the one extreme to the other by regular and similar gradations.

Thus the direct solar beam LI is so modified by reflection from the first plate, that, in its subsequent reflection from a similar plate, the intensity of the reflected portion depends not solely upon the angle of incidence as in its first reflection, but

also upon the position of the plate with respect to fixed objects in surrounding space.

The modification which the light undergoes in its first reflection, affects, it should be observed, the intensity only, not the direction of the subsequently reflected portion.

164. *Definitions.*

The pencil II′ thus modified by reflection, is said to be polarized; and the light constituting it is called *polarized light.* The plane passing through II′, and coinciding with LII′ the plane by reflection in which the light is polarized, is called *the plane of polarization;* and the light is said to be *polarized in this plane.** The plates MN and M′N′ are called the *polarizing* and *analyzing plates* respectively.

The plane of polarization of a polarized ray should be regarded as a plane constantly passing through the ray as it advances in space, and so related to it, that in any subsequent reflection of the ray, the quantity of light reflected depends, as we have seen in the preceding article, upon the position of this plane with respect to the plane in which the subsequent reflection takes place.†

* We distinguish the plane of first reflection from the plane of polarization, although the two are coincident; because, while the former is fixed, the latter must in general be regarded as movable.

† In the wave theory, the vibrations of the molecules are conceived to be made at right angles to the plane of polarization of the ray; and hence when it is said that a ray is polarized in a given plane, it is meant that the molecules vibrate at right angles to that plane.

The position of the plane of polarization remains the same relative to fixed objects in space, in all the future progress of the ray, unless the light suffer some further reflection or refraction. By subsequent reflection or refraction, its position may be changed.

We have supposed the light to fall upon the plate MN at an angle of 54° 35′. At any other incidence, the second reflected ray presents variations of intensity; but the difference between the maximum and minimum intensities is greatest when the angle of incidence on the first plate is 54° 35′. Hence this angle is called *the maximum polarizing angle*, or simply *the polarizing angle.*

165. *Laws of the polarization of light by reflection.*

1. All reflecting substances are capable of polarizing by reflection a greater or less portion of the light incident upon their surfaces; but the polarization varies in certain respects with the substance from which the light is reflected. The polarization which we are at present to consider is that which is produced by reflection from certain non-metallic substances whose indices of refraction do not vary much from 1.4. It is called *rectilinear* or *plane polarization*, and the light is called *rectilinearly polarized light*, or *plane polarized light.**

* The number of substances which have been found to rigorously polarize light rectilinearly, whatever the angle of incidence, is very small; as yet only four, viz. menilite, sulphate of the

2. The angle of polarization varies with the nature of the substance, and in such a manner that its tangent is always equal to the index of refraction of the substance, the index being taken with reference to the medium in which the light is reflected. Thus the indices of refraction for light passing from air into water, and from air into crown glass, being 1.33 and 1.53 respectively, we have, denoting the polarizing angles by p and p',

$$\text{tang } p = 1.33, \quad \text{and tang } p' = 1.53;$$

and hence $$p = 53°\ 11', \quad \text{and } p' = 56°\ 58'.$$

166. *Inferences from the second law.*

1. The indices of refraction of opaque substances can be determined from their polarizing angles.

2. The same medium cannot polarize rays of all colors at the same angle; and hence the intensity of the reflected polarized pencil can never be rigorously reduced to zero, except when the light is homogeneous.

3. When a ray of unpolarized light is incident on a transparent medium at the polarizing angle, the reflected and refracted portions make with each other a right angle. For denoting the polarizing

sesqui-oxide of iron, glycerine and alum cut perpendicular to the axis of the octahedron. But the polarization impressed upon light by reflection from many other substances, crown glass for example, is *sensibly* rectilinear except for certain angles of incidence within very narrow limits, especially when the intensity of the incident light is not great.

angle by p, the corresponding angle of refraction by r, and by n the index of refraction, we have, by the second law,

$$\text{tang } p = n, \quad \text{or } \sin p = n \cos p;$$

and by the ordinary law of refraction, we have

$$\sin p = n \sin r;$$

hence $$\cos p = \sin r,$$

or $$p + r = 90°.$$

4. When a pencil of unpolarized light is incident at the polarizing angle, on a transparent plate having parallel faces, not only is the portion reflected at the first surface wholly polarized, but that also which is reflected internally at the second surface. For, employing the same notation as above, we have

$$\sin p = n \sin r;$$

also, $$\sin p = \cos r;$$

and hence $$n \sin r = \cos r,$$

or $$\frac{\sin r}{\cos r} = \text{tang } r = \frac{1}{n}.$$

Here r is the angle of incidence on the second surface of the medium, and $\frac{1}{n}$ is the index of refraction for light passing from the medium into air. The angle of incidence on this surface is therefore equal to the polarizing angle for rays incident internally, and the light reflected from it is wholly polarized in its plane of reflection.

The light thus polarized, falling again upon the first surface, that part of it which is transmitted, suffers, as may be shown, no change in the position of its plane of polarization, and will therefore leave the surface polarized in the same plane as the first reflected pencil. Hence if a number of plates of glass or other transparent medium, having parallel faces, be laid one upon the other so as to form a *pile*, and a beam of light be incident upon the first surface at the polarizing angle, the portions reflected from the several surfaces, and finally transmitted through the first into the air, will leave the pile polarized in the same plane, and thus form a very intense polarized beam.

When the angle of incidence on the first plate is either greater or less than the polarizing angle, the second reflected beam (the angle at which it is reflected being equal to the polarizing angle) exhibits, as has already been remarked, variations in intensity as the second plate revolves, but never entirely disappears; the difference between the maximum and minimum intensities being the less, the more the angle of first incidence differs from the polarizing angle. The light is in this case said to be *partially polarized;* by which it is meant that the incident light is resolved into two portions, one of which is polarized, while the other is entirely unaffected, or in the condition of common light. It has been found that by repeated reflec-

tions at any angles whatever, a beam of light may be wholly polarized, provided all the reflections take place in the same plane.

Light radiated from imperfectly polished surfaces is in general partially polarized.

Solar light, diffused by reflection from the particles of the atmosphere, is polarized; and as the reflection takes place at all the angles, the amount polarized varies between certain maximum and minimum values.

167. *The laws of the reflection of polarized light.* When a beam of polarized light is reflected from any surface, transparent or opaque, the direction of the reflected rays is precisely the same as in the case of common light. The laws now to be considered relate to the intensity of the reflected light, and the position of the plane of polarization after reflection. The simplest case is that in which the polarized pencil falls on the reflecting surface at a constant angle of incidence equal to its polarizing angle, and the surface revolves about the pencil as an axis. These conditions exist in the experiment of Art. 163.

Denoting, as in that article, the variable and maximum intensities of the reflected beam by i and a respectively, and by d the variable angle which the plane of first reflection, or the plane of

polarization, makes with the plane of second reflection, we have for the extreme cases the relation

$$i = a \cos^2 d;$$

since for $d = 0$ or $d = 180°$,

this equation gives $i = a$,

and for $d = 90°$ or $d = 270°$,

it gives $i = 0$.

This equation, which is thus evidently true for the extreme cases, and which was at first assumed as the simplest that would satisfy them, has since been verified in intermediate positions of the planes of reflection, and may be considered generally true. It is also a deduction from the wave theory.

A singular deduction from this equation, is that we may regard a ray of common light, so far as its intensity is concerned, as composed of two polarized rays of equal intensity, whose planes of polarization are at right angles to each other. For supposing a ray thus compounded to be incident on a reflecting surface, and denoting the maximum intensity of the reflected portions of the simple rays composing it by a, and their variable intensities by i and i' respectively; also representing by d the angle which the plane of polarization of the first simple ray makes with the plane of reflection, we have

$$i = a \cos^2 d.$$

Also since the angle which the plane of polarization

of the second simple ray makes with the plane of reflection is $(90° - d)$, we have

$$i' = a \cos^2(90° - d) = a \sin^2 d;$$

and hence

$$i + i' = a (\sin^2 d + \cos^2 d) = a.$$

That is, the total intensity of the compound reflected ray is the same for every value of d, or for every position assumed by the reflecting plane as it revolves about the compound incident ray at a constant angle; which is the characteristic property of common or unpolarized light. Any two such rays as those composing the compound incident ray, are said to be *oppositely polarized.*

For the general case in which the polarized ray falls upon the reflecting surface at any angle whatever, a formula which gives the intensity of the reflected ray has been deduced from the theory. It is much more complicated than that which we have been considering.

168. *Change of position of the plane of polarization of the reflected ray.*

When a beam of polarized light is incident upon a reflecting surface, at any angle whatever, the reflected portion is still polarized, but its plane of polarization in general suffers a change of position. The following are the results of experiment:

1. The movement of the plane of polarization is always directed towards the plane of reflection.

2. When the angle of incidence is zero, the plane of polarization suffers no change.

3. When the angle of incidence is equal to the polarizing angle, the plane of polarization is brought to a coincidence with the plane of reflection, whatever may have been the position of the plane of polarization of the incident ray.

4. When the incident ray is polarized in the plane of incidence, the position of the plane of polarization remains unchanged by reflection, whatever the angle of incidence.

5. It also remains unchanged by reflection, when the incident ray is polarized in a plane perpendicular to the plane of incidence.

The law according to which these changes take place, is expressed by the formula

$$\operatorname{tang} b' = \frac{\cos(i + r)}{\cos(i - r)} \operatorname{tang} b;$$

in which b denotes the angle made by the plane of polarization of the incident ray, with the plane of incidence; b', the angle which the plane of polarization of the reflected ray makes with that plane, or with the plane of reflection; and i and r, the angles of incidence and refraction, connected by the relation

$$\sin i = n \sin r,$$

n being the index of refraction of the substance of the reflector.

From this formula, we can immediately deduce the experimental results just given.

1. Since each of the angles i and r is less than $90°$, we have constantly

$$\cos (i + r) < \cos (i - r),$$

and hence $b' < b$; that is, by reflection, the plane of polarization is made to approach the plane of reflection.

2. When $i = 0$, we have $b' = b$.

3. When $(i + r) = 90°$, that is, when the angle of incidence is equal to the polarizing angle, we have $b' = 0$, whatever the value of b.

4. When $b = 0$, we have $b' = 0$, whatever the value of i.

5. When $b = 90°$, we have $b' = 90°$.

169. *Polarization of light by ordinary refraction.* When a beam of common light is incident upon a plate of glass, the portion of it which is refracted is found to be partially polarized in a plane perpendicular to the plane of refraction. The quantity of polarized light in the refracted beam varies with the angle of incidence, and is nothing when that angle is zero. The light thus polarized by refraction is intimately related to that part of the same incident beam which is polarized by reflection. The general law may be thus enunciated: When an unpolarized ray is partly reflected at, and partly transmitted through, a transparent surface, the reflected and transmitted portions contain equal

quantities of polarized light, and have their planes of polarization at right angles to each other.

If the light be incident on a pile of plates, at the polarizing angle, the pencil transmitted through the first surface of the first plate will evidently contain its maximum of polarized light; and as this maximum is equal to the light which the surface is capable of polarizing by reflection, and as the light polarized by reflection from any surface is always much less than half the incident light, the transmitted portion will contain a certain amount of unpolarized light. At the second surface of the first plate, that part of the transmitted light which is polarized being incident at the polarizing angle, and having its plane of polarization perpendicular to the plane of incidence, will be wholly transmitted; and as its plane of polarization, as may be shown by experiment, remains unchanged, it will also be capable of transmission undiminished, supposing no absorption, through the remaining surfaces however numerous the plates. The unpolarized part will be divided, in the same manner as the light incident on the first surface, into reflected and refracted portions, and the latter will comport itself in the same manner as the light refracted by the first surface. By employing a sufficient number of plates, the unpolarized portion may be sensibly reduced to zero.

170. *Polarization by double refraction.*

When a pencil of unpolarized light is incident at right angles upon a crystal of one axis, a crystal of carbonate of lime, for example, the two equally intense emergent pencils into which the transmitted portion is divided, are found to be wholly polarized rectilinearly: the ordinary pencil, in a plane parallel to the principal section of the crystal; and the extraordinary pencil, in a plane at right angles to it.

If, instead of common light, a pencil be employed which has been polarized by any process whatever, then,

1. When the plane of polarization of the incident pencil is parallel to the plane of principal section of the crystal, the transmitted pencil is found to suffer ordinary refraction only.

2. When its plane of polarization is perpendicular to the plane of principal section of the crystal, it is found to suffer extraordinary refraction only.

3. In all intermediate positions of the plane of polarization, it is found to be refracted doubly: the ordinary pencil being the brighter, if the plane of primitive polarization is nearer the position of parallelism than of perpendicularity; and the extraordinary pencil the brighter, if the plane of polarization is nearer the position of perpendicularity than that of parallelism; both pencils being equally bright, when the plane of polarization is equally distant from these extreme positions.

If we denote by Po the intensity of that portion of the transmitted pencil which suffers ordinary refraction, by Pe the intensity of the portion which suffers extraordinary refraction, by A the intensity of the light which enters the crystal, and by a the angle which the plane of polarization of the ray makes with the principal section of the crystal, the exact relations between these several quantities will be expressed by the following equations:

$$Po = A \cos^2 a, \quad Pe = A \sin^2 a.$$

These equations, which have been satisfactorily verified, are evidently true for the extreme cases, and also for the case in which $a = 45°$. Admitting no loss of light, the sum of the intensities of the two pencils should be equal to the intensity of the light which enters the crystal; and in accordance with this, we have

$$Po + Pe = (\cos^2 a + \sin^2 a) A = A.$$

SUPERIMPOSED CRYSTALS.

If the two emergent pencils into which a beam of common light is separated by transmission through a crystal of carbonate of lime, be received upon a second crystal of the same mineral, so placed that its edges may be parallel to the corresponding edges of the first, and the one crystal be made to revolve on the face of the other; then,

1. When the planes of principal section of the two crystals coincide, neither of the pencils is divided by its transmission through the second

crystal; O the ordinary pencil suffering ordinary refraction only, and E the extraordinary pencil extraordinary refraction only.

2. When the planes of principal section are at right angles to each other, the pencils are again transmitted undivided; in this case O suffering extraordinary refraction, and E ordinary refraction.

3. In every other position assumed by the crystal, both O and E are refracted doubly, and four distinct pencils emerge from the second surface of the second crystal. When the planes of principal section make with each other an angle of 45°, these four pencils are equally intense; in all other cases their intensities are unequal, the light of each of the equally intense pencils O and E being divided unequally between the corresponding refracted pencils.

The two pencils ordinarily refracted by the second crystal are found to have their planes of polarization parallel to its principal section, the two extraordinarily refracted to have theirs at right angles to it.

Admitting that none of the light which enters the first crystal is lost by absorption or reflection, the intensities of the several pencils, in the case just considered, will be given by the following equations:

$$\left.\begin{aligned} Oo &= \tfrac{1}{2}A\cos^2 a, \\ Oe &= \tfrac{1}{2}A\sin^2 a, \end{aligned}\right\} \qquad \left.\begin{aligned} Ee &= \tfrac{1}{2}A\sin^2(90-a) = \tfrac{1}{2}A\cos^2 a, \\ Eo &= \tfrac{1}{2}A\cos^2(90-a) = \tfrac{1}{2}A\sin^2 a; \end{aligned}\right\}$$

in which Oo denotes the intensity of the pencil ordinarily refracted by both crystals, Oe the intensity of the pencil ordinarily refracted by the first and extraordinarily by the second, Eo the intensity of the pencil refracted extraordinarily by the first and ordinarily by the second, Ee the intensity of the pencil refracted extraordinarily by both; also a the angle made by the principal sections of the two crystals, and A the intensity of the light which enters the first crystal.

On the supposition above made, that all the light which enters the first crystal emerges from the second, the sum of the intensities of the four pencils should be equal to A; and accordingly we find, by a simple reduction,

$$Oo + Oe + Eo + Ee = A.$$

171. *Polarization by absorption.*

When a beam of common light is incident perpendicularly upon a plate of tourmaline about the twentieth of an inch thick, formed by cutting a crystal of that mineral in planes parallel to its axis, the emergent portion is found to be polarized in a plane perpendicular to the principal section of the plate.

If, instead of unpolarized light, we employ a pencil previously polarized in any way whatever, when its plane of polarization is parallel to the

principal section of the crystal, the light is completely absorbed; but if the pencil be polarized in any other plane, a greater or less portion of it is transmitted, the transmitted portion having its maximum when the plane of polarization is perpendicular to the principal section. We hence derive the following analysis of the process by which a tourmaline plate polarizes common light:

Doubly refracting crystals, like other transparent media, absorb more or less of the light which penetrates them; and the absorption of the polarized pencils varies with the position of the planes of polarization with respect to the axis of the crystal. Thus, in a tourmaline, the pencil polarized in a plane of principal section is absorbed much more rapidly than the pencil polarized in a plane perpendicular to it. Now when a pencil of unpolarized light falls upon a plate of this crystal, it is resolved into two portions, one of which is polarized in the plane of principal section, and the other in a plane at right angles to it. Consequently the former of these is absorbed much more rapidly than the latter; and when the plate is sufficiently thick, the latter portion alone is sensible in the transmitted light. From this explanation, it appears that the intensity of the emergent pencil is always much less than the intensity of the incident light.

The action of tourmaline plates on light is strikingly illustrated in the following experiment:

When one of these plates is interposed between a luminous object and the eye, at right angles to the line of vision, the object appears equally distinct in every position assumed by the axis as the plate revolves in its own plane; but if, keeping the plate in a fixed position, its axis vertical for example, a second plate be interposed parallel to the first, and turned round slowly in its own plane, then the brightness of the object is found to vary with the angle made by the axes of the plates: thus when the axes are at right angles to each other, the object is scarcely visible; but as the plate revolves, it is seen more and more distinctly, till, when the axes are parallel to each other, it appears with its maximum brightness. The plate continuing to revolve, the brightness diminishes by the same gradations by which it before increased, till the axes are again at right angles, when it is at the minimum. Through the remaining 180°, similar appearances are presented.

Instruments for polarizing light are called *polarizers;* those for detecting polarized light and determining its plane of polarization, *polariscopes* or *analyzers.*

In the instrument known as the apparatus of NOREMBERG, the essential parts of which are the plates MN, M′N′, figure 126, a polarizer and an analyzer are combined. But the simplest instrument by which light may be both polarized and

analyzed, consists of two plates of tourmaline, as described above, either of which may be revolved in its own plane parallel to the other. A prism of carbonate lime is used as an analyzer in several optical instruments; but the most valuable analyzer, especially on account of its transparency, is that called NICOL'S prism. It is made by cutting a crystal of carbonate of lime, whose length is about three times its thickness, into two parts, in a plane passing through the vertices of the obtuse angles, perpendicular to the plane of the longer diagonals of its rhombic bases, and then re-uniting the parts in their former positions by Canada balsam. The index of refraction of the balsam is less than the ordinary index of the carbonate of lime, but greater than the extraordinary index; and the result is, that the ordinary ray suffers total reflection, and is thrown to the side of the crystal, and the extraordinary ray is transmitted.

These instruments are reciprocal in their properties; a polarizer may be used as an analyzer, an analyzer as a polarizer.

INTERFERENCE OF POLARIZED LIGHT.

172. The subject of the interference of polarized light was first investigated by ARAGO and FRESNEL. The most important results of their experiments are embodied in the following laws:

1°. Two rays, polarized in the same plane, interfere with each other in precisely the same manner as unpolarized rays, so that the phenomena of interference in the two species of light are absolutely the same.

2°. Two rays oppositely polarized, that is, in planes at right angles to each other, do not interfere, in the very same circumstances in which unpolarized rays would interfere so as to destroy each other.

The first law may be verified by repeating, with polarized light, any of the experiments on diffraction described in Chapter VIII, Part I. The results will be found to be identical with those obtained when common light is used.

The verification of the second law is more difficult. One of the simplest methods of affecting it, consists in cutting a plate of tourmaline, which has been reduced to a uniform thickness, into two parts, and placing the parts in the paths of the interfering rays. The intensity of the fringes is found to depend on the relative position of the axes of the tourmaline; the intensity being greatest when the axes are parallel to each other, and the parallel to each other, and the two rays are consequently polarized in the same plane, and diminishing as the axes become mutually inclined, the stripes entirely disappearing when they make with each other a right angle.

173. *Of the colors exhibited by crystallized plates when exposed to polarized light, and of the polarized rings which surround their optic axes.*

When a ray of polarized light is transmitted through a thin plate of any doubly refracting substance, and each of the two oppositely polarized rays thus produced is again resolved into two other rays also oppositely polarized, the resulting phenomena are of the most novel and interesting character. The apparatus for exhibiting these effects consists essentially of a polarizer, the doubly refracting plate, and an analyzer. For the polarizer we may employ a reflector of glass, or a plate of tourmaline; for the crystalline plate, a film of mica or of some other crystal; and for the analyzer, a glass reflector, a plate of tourmaline, or a doubly refracting prism.

Let B [Fig. 128] represent the polarizer; D the crystalline plate, cut parallel to its optic axes, and so placed as to receive the polarized beam at right angles; and G the analyzer, in the present case supposed to be a doubly refracting prism, having its lateral faces parallel to its optic axis, and placed at right angles to the axis AO′ of the apparatus.

1. Let the crystalline plate be removed from the apparatus: then the polarized ray C, falling directly upon the prism, suffers double refraction, and the eye, placed behind the prism, sees two images, always white, and in general unequally intense, one of which disappears, when, by the

revolution of the prism, its principal section becomes either parallel or perpendicular to the plane of primitive polarization.

2. Let the prism be removed, the plate and polarizer being retained: then the eye, placed at O′, perceives in all cases a single white image; the separation of the rays, which actually takes place, being, on account of the thinness of the plate, insensible.

3. Let the several pieces of the apparatus occupy their respective places: then when the principal section of the plate is either parallel or perpendicular to the plane of primitive polarization, the phenomena are precisely the same as if the plate were not interposed; in general, two white images being perceived, which undergo certain variations of intensity as the prism revolves. But in every other position assumed by the plate as it revolves about the ray as an axis, these two images, visible as before, are not white, but colored; the tint of each being uniform, and the two tints complementary to each other.

. The colors of the two images change with the thickness of the plate, the brilliancy in general increasing as the thickness diminishes. As the thickness of the plate increases, a limit is reached, above which the light emergent from the prism ceases to be colored: this limit varies with the nature of the crystal; in mica, for example, it is about $\frac{1}{30}$ of an inch.

We have supposed the thickness of the plate to be the same throughout: when this is not the case, each of the colored images exhibits a greater or less number of different tints.

174. *Explanation.*

When the principal section of the crystalline plate is neither parallel nor perpendicular to the plane of primitive polarization, the polarized ray C will be separated into the two parallel rays O and E, polarized, the one in the principal section of the plate, the other in a plane at right angles to it. These two rays, or systems of undulations, having passed through the plate with unequal velocities (a deduction from the wave theory, as we shall presently see), the one will consequently fall behind the other; but on account of the thinness of the plate, the retardation will not exceed a few undulations; and, for the same reason, the rays will emerge almost superimposed. Thus related, they will fall upon the prism, and in general again suffer double refraction: the ordinary ray O being resolved into the two rays O*o*, O*e*, and the extraordinary ray E into the rays E*o*, E*e*; O*o*, E*o* being polarized in the plane of principal section of the prism, and E*e*, O*e* in a plane perpendicular to it. The two rays, or systems of undulations O*o*, E*o* being superimposed, having a common plane of polarization, and being retarded the one relative to the other by a

few undulations and parts of an undulation only, will interfere, and for similar reasons so also will the rays E*e*, O*e*. But again having recourse to the wave theory, when the light employed is homogeneous, the two images thus produced will, by the law of the conservation of living forces, be complementary in intensity*; consequently when it is compound, these images will be complementary in color.

175. That the ordinary and extraordinary rays are, as we have supposed, transmitted through a doubly refracting substance with unequal velocities, may be inferred from the equation [Art. 137],

$$\frac{\sin \mathrm{I}}{\sin \mathrm{R}} = \frac{v}{v'} = n.$$

In positive crystals the ordinary ray moves with the greater velocity; in negative crystals, the extraordinary ray.

176. The foregoing experiment may be varied, so as to produce an exceedingly interesting result, by causing the polarized pencil to fall upon the plate in the direction of its optic axis, and giving

* How this takes place, may be shown by resolving, by means of the parallelogram of velocities, a single vibration, the path of which is represented by a straight line, into two vibrations (those of the molecules of O and E) at right angles to each other, and each of these into two (those of the molecules of O_o, O_e, E_o, E_e) also at right angles. It will be seen, that of these four vibrations, two are made in the same sense, two in opposite senses; the latter case being the same as that of article 140, in which the ordinary conditions of interference are reversed.

to the eye such a position that it shall receive the rays which traverse the plate in directions inclined to the axis at small angles.

Let the crystal from which the plate is cut be uniaxal, and, for the sake of simplicity, suppose the sections to be perpendicular to the axis. Suppose, also, the analyzer to be a tourmaline plate, and let the eye be placed almost in contact with it.

Then when the principal section of the tourmaline is parallel to the plane of primitive polarization, the observer will perceive a series of concentric rings [Fig. 129] encircling a black spot in the direction of the optic axis of the crystalline plate, and traversed by a black rectangular cross. The arms of the cross meet at the common centre, and one of them lies in the plane of primitive polarization.

When the incident light is homogeneous, the rings, separated by dark intervals, are of the same color as the light employed. Their diameters vary with the color of the light; in general increasing with the refrangibility. When the incident light is white, the system consists of iris-colored rings, the result of the superposition of all the homogeneous systems.

When the axis of the tourmaline is perpendicular to the plane of primitive polarization, a new system of rings [Fig. 130] takes the place of the former system, the colors of which are complementary to those of the former. The cross is in this case of

the same color as the incident light: white, when white light is employed; red, when the light used is red.

When the change in the position of the tourmaline is made by revolving it slowly in its own plane, the first system of rings may be seen to pass gradually into the second.

177. *Explanation.*

We will consider the case in which the principal section of the tourmaline is parallel to the plane of primitive polarization. Let O′ [Fig. 131] be the place of the eye, and MM′ the second surface of the plate. The observer will see the rings by means of a cone of rays, which has its vertex at O′, its base II′ in the plane MM′, and its axis AO′ coincident with the axis of the plate. The rays near the axis AO′, traversing the plate in the direction of its optic axis, undergo no change whatever, and, entering the tourmaline with their planes of polarization parallel to its principal section, are consequently absorbed: hence the black spot in the direction of the axis.

Again: let II′BB′ [Fig. 132] represent the base of the cone of rays, II′ the plane of primitive polarization, and BB′ a plane perpendicular to it. Any plane drawn through A, perpendicular to the face IBI′B′, is evidently a principal section of the plate. Hence the rays which traverse the plate in the plane II′ have their planes of polarization coinci-

dent with a principal section of the plate, and therefore suffer ordinary refraction only, without undergoing 255 any change in the position of their plane of polarization. Hence, also, the rays transmitted in the plane BB′ have their planes of polarization perpendicular to a principal section of the plate, and suffer extraordinary refraction only, without any change in the position of their plane of polarization. These two sets of rays being thus polarized in planes parallel to the principal section of the tourmaline, are therefore extinguished; and hence is produced the appearance of the black cross.

The formation of the rings is explained in the following manner: A ray transmitted in any intermediate plane of principal section, as CD for example, being polarized in a plane neither parallel nor perpendicular to CD, will be resolved into two other rays, the ordinary and extraordinary, having their planes of polarization respectively coincident with CD and perpendicular to it, and the one falling a few semi-undulations behind the other. By the action of the tourmaline, each of these will be separated into two others, having their planes of polarization parallel and perpendicular, respectively, to its principal section. Of the four rays thus produced, the two parallel to the principal section will be absorbed, while the remaining two will interfere. Points of maximum and minimum intensity will thus be produced, which, as every-

thing takes place symmetrically about the axis A, will be arranged in circles; and hence will result, in homogeneous light, rings alternately colored and dark; and in compound light, iris-colored rings. When the principal section of the tourmaline is perpendicular to the plane of primitive polarization, the system of rings should evidently be complementary to the one just considered. With a doubly refracting prism as an analyzer, both systems of rings may be seen at the same time.

178. In biaxal crystals, cut perpendicularly to the line bisecting the optic axes, two systems of rings are observed, one about each axis. These rings are usually of an oval form, and crossed by dark bars or brushes.

When, instead of plates of crystallized minerals, we interpose between the polarizer and analyzer certain other substances, as unannealed glass, the crystalline lens of fishes, compressed glass, animal jelly, pieces of quills, etc., tints and systems of colored rings are produced, similar to those described above, indicating the existence of a doubly refracting structure. Indeed, whenever there exists in a transparent body the least tendency to a peculiar arrangement of the particles, it may be detected, and the nature of it, partially at least, determined by transmitting through the body a polarized ray. The importance of this fact, in relation to our know-

ledge of the molecular constitution of bodies, is obvious.

179. *Elliptical and circular polarization.*

When, in the experiment described in article 163, a metallic mirror is substituted for the glass plate MN, figure 126, it is found, that although as the analyzing plate revolves, the intensity of the reflected light suffers, as before, a variation, it is never reduced to zero when the planes of first and second reflection are at right angles to each other, whatever the angle of first reflection, but only to a certain minimum, as if the light were only partially polarized by the first reflection. But if a thin doubly refracting plate be interposed, the light reflected from the second plate exhibits tints different from those which would be produced by partially polarized light under the same treatment. By reflection from the metallic plate, the light has received a polarization distinct from rectilinear or plane polarization. Indeed it is found to consist of two pencils unequally intense, and differing in phase, the one polarized in the plane of reflection, the other in a plane at right angles to it. The same kind of polarization is impressed upon light by reflection from nearly all non-metallic substances. [See note to article 165]. Doctor Brewster, who first detected this kind of polarization, called it *elliptical polarization;* and the light thus polarized, *elliptically polarized light.* These terms were em-

ployed by him without any reference to theoretical considerations.

The term *circular polarization* is used to designate a modification of light similar to elliptical polarization, and the light thus modified is called *circularly polarized light.* This species of light, when examined by an analyzer, suffers no variation in intensity as the instrument revolves, in this respect resembling common light. But it differs essentially from common light, because if a doubly refracting plate be interposed, the light reflected or transmitted by the analyzer appears colored. The color, it may be added, is not the same as would be produced by *rectilinearly polarized light.*

The conclusion from the foregoing is, that by reflection, light in general receives elliptical polarization, and that rectilinear or plane polarization and circular polarization should be regarded as exceptions.

180. *Rotation of the plane of polarization.*

We have seen, that in general, when a ray of polarized light is transmitted through a plate of Iceland spar, or other uniaxal crystal, in the direction of its axis, *the position of its plane of polarization remains unchanged;* so that when the light is analyzed by a tourmaline plate, having its axis in the plane of primitive polarization, no light is transmitted through the plate. To this there are certain exceptions; thus, if instead of a plate of

Iceland spar, we employ one of rock-crystal, then in the same circumstances as above (the ray, for the sake of simplicity, being supposed homogeneous), more or less light will pass through the analyzer; and to extinguish it, the tourmaline must be revolved through a certain angle. By the transmission of the ray through the plate, therefore, the position of the plane of polarization is in this case changed. In some specimens of quartz, this rotation of the plane of polarization takes place to the right of its primitive position; in others to the left of it; and the crystals are termed right-handed or left-handed, according as they produce the one or the other of these effects.

This phenomenon has been found by BIOT to take place according to the following laws:

1°. The action of different plates of the same crystal is always exerted in the same direction, and is proportional to the thickness of the plate.

2°. The effect of two or more plates superimposed, is equal to the sum or difference of the partial effects, according as they act in the same or opposite directions.

3°. The action of a plate on a ray is the more energetic, the greater the refrangibility of the ray. For a given plate, the arc through which the plane

of polarization is made to rotate is inversely as the square of the length of the wave.*

From the third law, it follows, that if the polarized beam consist of white light, the light viewed by the analyzer, after its emergence from the plate, will be colored. For supposing the analyzer, for example, to be a reflector, it will reflect the rays of different colors in different proportions, depending upon the position of their planes of polarization with respect to the plane of reflection; and hence will result a compound tint, which will vary with the position of the reflector.

The property of producing rotation of the plane of polarization is not peculiar to quartz. It is found in other solids, and also in certain liquids. Among the solids are crystals of cinnabar, sulphate of strychnine, the chlorate and bromate of soda; among the liquids, oil of turpentine, oil of lemon, solutions of sugar in water, solution of camphor in alcohol. It is also found in the vapor of spirits of turpentine.

This property may be employed in determining molecular changes in bodies which other means do not indicate, as, for example, the transformations which take place in the juices of plants during the period of growth.

The power to rotate the plane of polarization can be imparted to various transparent substances,

* More recent experiments seem to indicate that this law is only approximately true.

both solid and liquid, as flint glass, bisulphide of carbon, water, alcohol, ether, etc., by subjecting them to the action of a powerful electro-magnet; the substance being placed between the poles, and the polarized ray transmitted in the direction of the line which joins them.

ON THE APPLICATION OF THE WAVE THEORY TO THE EXPLANATION OF THE PHENOMENA OF POLARIZED LIGHT.

181. The numerous and complicated phenomena of polarized light are susceptible of a complete explanation on the principles of the wave theory; but it is not within the scope of the present treatise to consider this portion of the theory of light; it will be found amply developed in the works of HERSCHEL, BILLET, JAMIN and others. The design of the following articles is barely to furnish the student with a few general ideas on the subject.

182. *Rectilinearly polarized light.*

Through a straight line, representing the direction of a ray of light, conceive two planes to be drawn at right angles to each other; and, to aid the conception, let one be supposed horizontal, the other being vertical. Then let the ethereal molecules, by the motion of which the light is produced, be supposed to vibrate in the horizontal plane *only*, at right angles to the direction of the ray, and hence perpendicular to the vertical plane; in the same manner as, in figure 99, we have supposed

the molecules to vibrate in the plane of the paper at right angles to *aw*. A ray thus constituted, evidently answers the essential condition of a rectilinearly polarized ray. We assume, therefore, that in such a ray, the molecules vibrate constantly in the same plane. As to the position of this plane, certain results indicate that it should be considered perpendicular to the plane of polarization of the ray. Thus, in the wave, theory, *a rectilinearly polarized ray is one in which the molecules vibrate constantly in right lines perpendicular to the same plane, that plane being the plane of polarization of the ray.* This conception, it will be perceived, results immediately from the principle of transverse vibrations.

183. *Unpolarized light.*

Now if we conceive the plane of vibration of a rectilinearly polarized ray to revolve about the direction of the ray as an axis, with great rapidity, the particles continuing to vibrate in the plane as before, we shall evidently have a ray which will be related in precisely the same manner to all the parts of surrounding space, and, consequently, one which answers the essential conditions of a ray of common or unpolarized light. Thus a ray of common light may be supposed to be one in which the plane of vibration is constantly, or at least at very short intervals of rest, revolving rapidly about the direction of the ray.

This conception has been verified by DOVE. He gave to a Nicol's prism a rapid motion of rotation, and then let fall upon it a pencil of rectilinearly polarized light in the direction parallel to the axis of motion. No trace of polarization could be perceived in the emergent light. But when the light of the electric spark was used, the polarization was immediately recognized, the duration of the light being too brief to permit the prism, and with it the direction of the vibrations, to revolve to the extent requisite to produce the sensation of natural light.

184. *Polarization.*

According to the foregoing views, the rectilinear polarization of a ray of common light consists in the resolution of the vibrations of the molecules into two distinct sets of vibrations, made perpendicular respectively to two planes at right angles to each other. Thus when a ray of common light falls upon a plate of glass at the polarizing angle, the forces brought into play are such, that after incidence, the vibrations are made partly in the plane of reflection, and partly at right angles to it; the reflected undulations consisting of those which are made at right angles to this plane.

185. *Interference; the several kinds of polarized light.*

The interference of two rays of unpolarized light, having a common origin, is an obvious consequence of the interference of two rays polarized in the

same plane; and the interference of two rays thus polarized, is the case already considered in article 126. The non-interference of two rays polarized in planes at right angles to each other [Art. 172, 2°], is also an obvious consequence of the theory. It is also a mathematical deduction from the theory, that while the intensity of the resulting light, in the case of two rectilinearly polarized rays, whose planes of polarization are at right angles to each other, is constantly equal to the sum of the intensities of the separate rays, the nature of the vibration varies with the phase in which the two rays meet: thus it is found, that when the rays differ in phase by an exact number of semi-undulations, the resulting vibrations will still be rectilinear; that when they differ by an odd multiple of a quarter of an undulation, they will be circular; and that in all other cases, they will be elliptical, the axis of the ellipse retaining constantly the same direction. When the molecules thus vibrate in an ellipse, the light is said to be *elliptically polarized;* when in a circle, to be *circularly polarized.*

But this elliptically polarized light of the theory is found to be identical in its properties with the elliptically polarized light of Doctor Brewster. We conclude, then, that the most general form of polarized light is that in which the molecules of the ether vibrate in ellipses whose planes are per-

pendicular to the direction of the light, and whose axes have a fixed direction in space; the eccentricity of the ellipses varying in different cases from zero to unity, and the vibrations in the extreme cases being thus either circular or rectilinear. The conditions of a ray of common light are evidently answered by a ray such as would be produced by the revolution of an elliptically polarized ray about its line of direction. The revolution need not be incessant, but may be interrupted by very short intervals of rest; but each complete revolution must take place in so short a time that the effect shall be sensibly the same as if the vibrations were made in all azimuths at the same instant.

186. *Double refraction.*

The relation between the particles of a crystal, and the molecules of the ether within it, is supposed to be such that in respect to elasticity, the condition of the ether is determined by that of the crystal. But in a doubly refracting crystal, the elasticity is found to vary with the direction in which force is applied; and hence the elastic force of the ether within such a crystal must, in general, be supposed different in different directions. From this hypothesis, combined with the principle of transverse vibrations, has been deduced a complete explanation of the phenomena of double refraction.

P. 81.
234

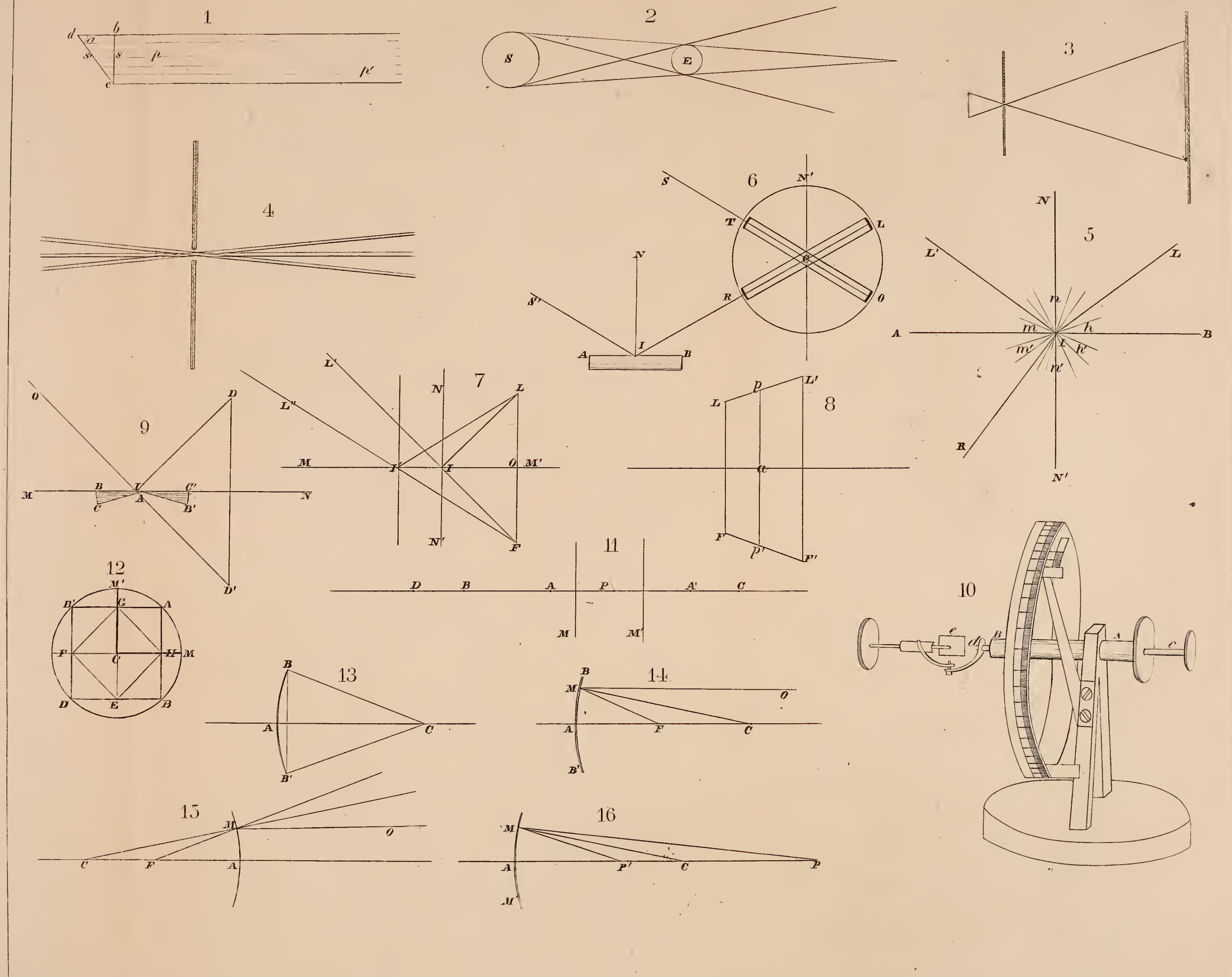
Define rectilinear polarization, + p. 255

Polarization by reflection, in same plane as of incidence.

" " refraction " right ang. to plane of incidence.

" " double " " rectilinearly polariz. o. plane parr. to princl. sect.

" " absorption. e. " perpen. " " perpendic. to the principal sect. of plate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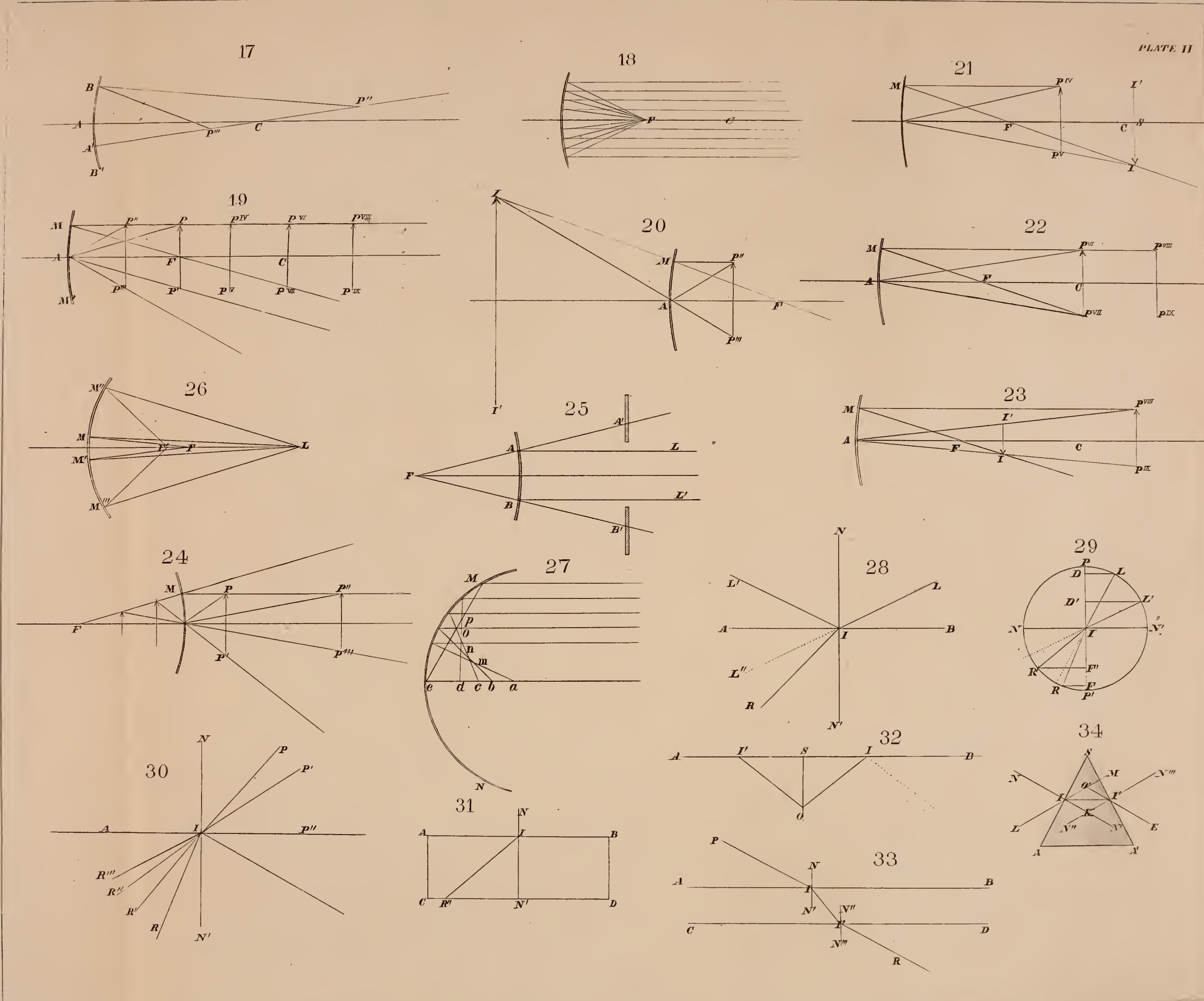
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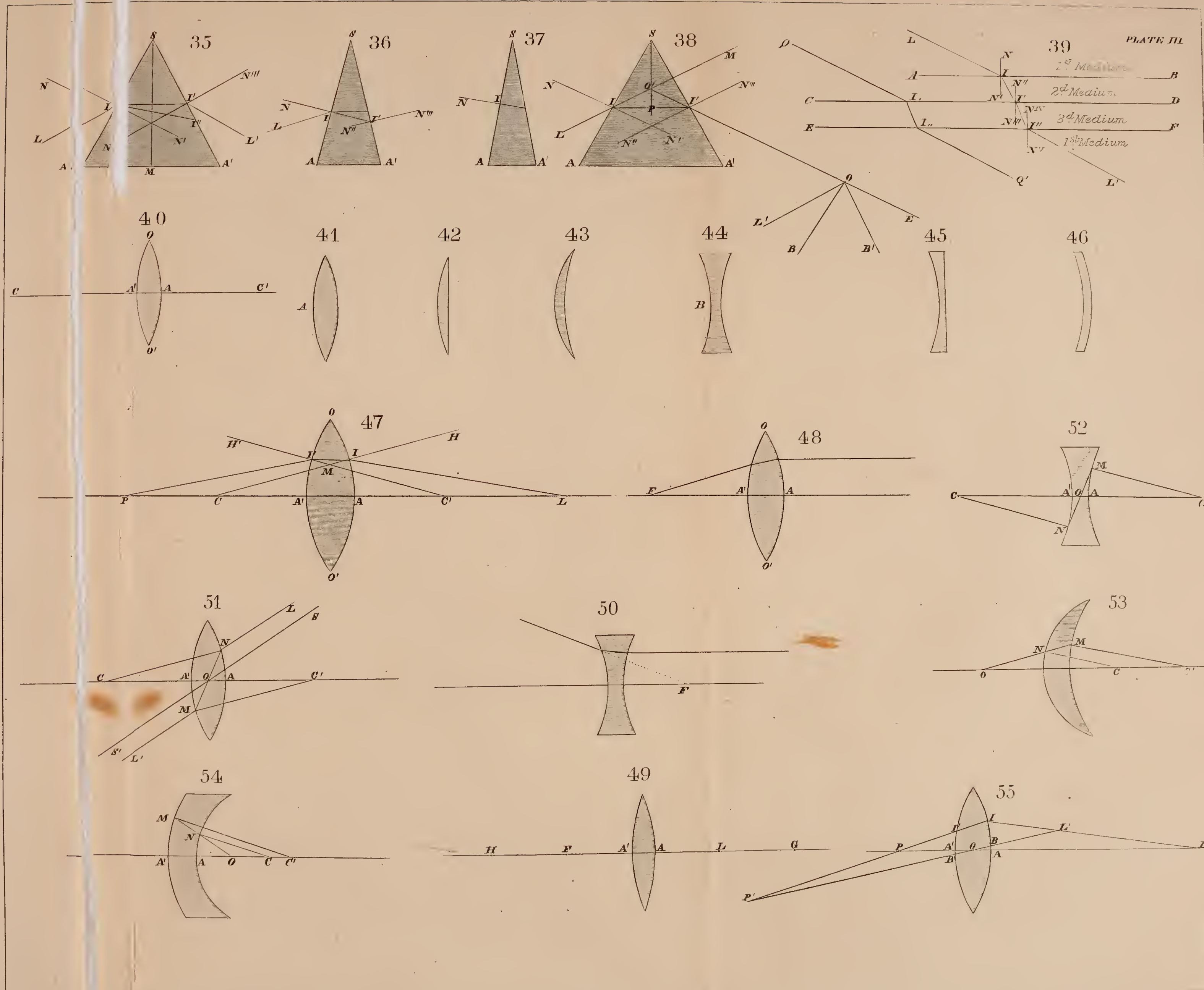
PLATE III.
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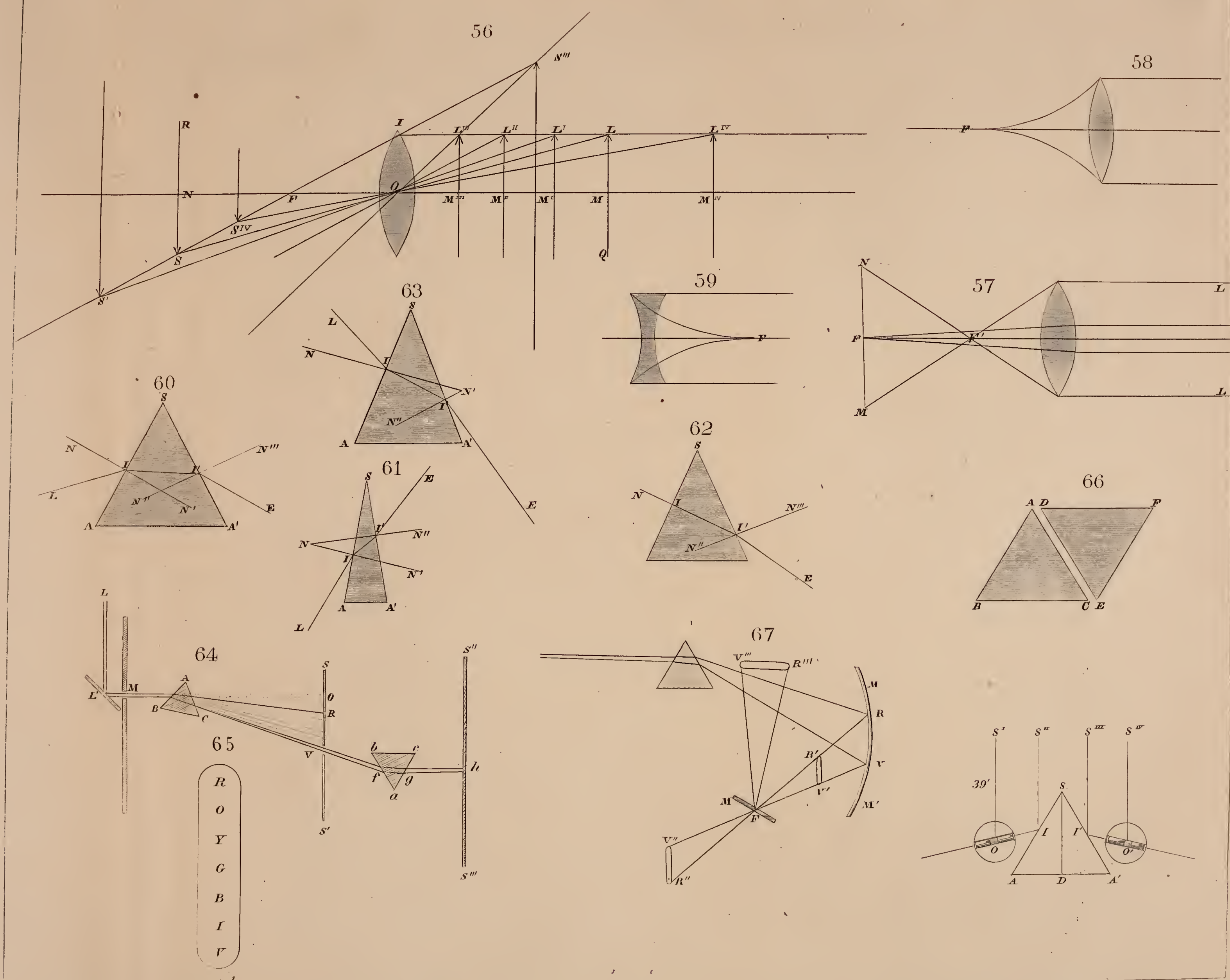
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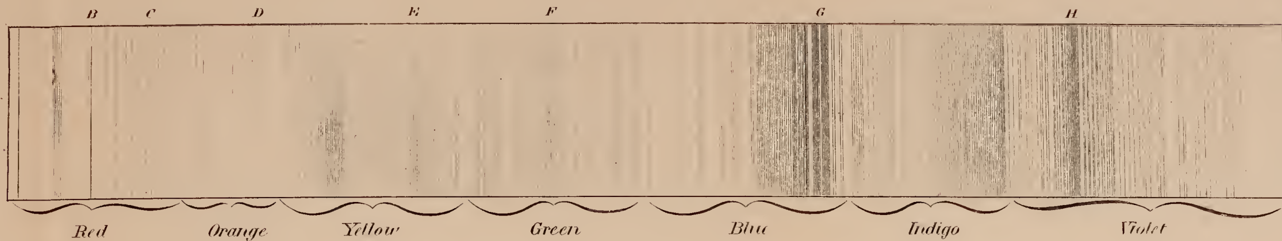

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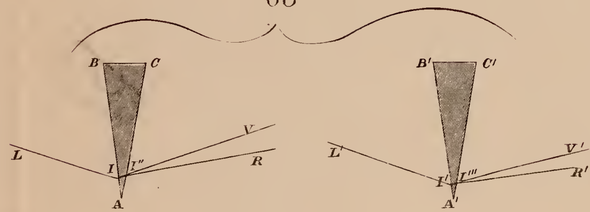

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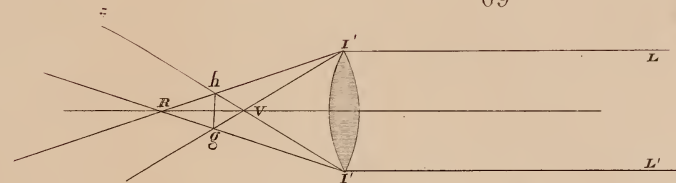

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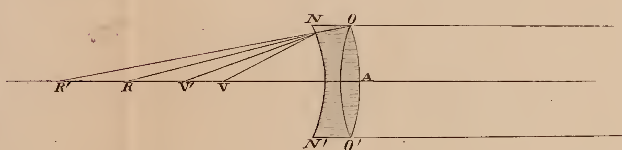

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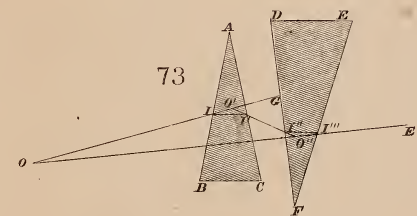

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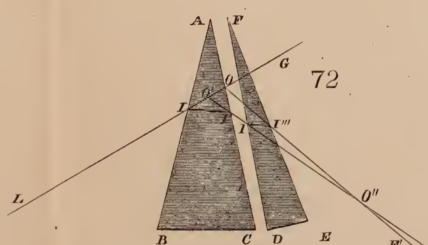

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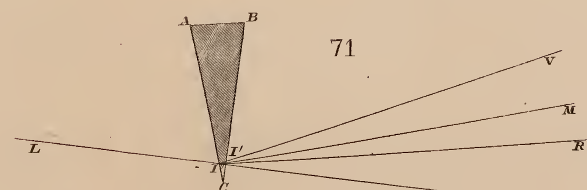
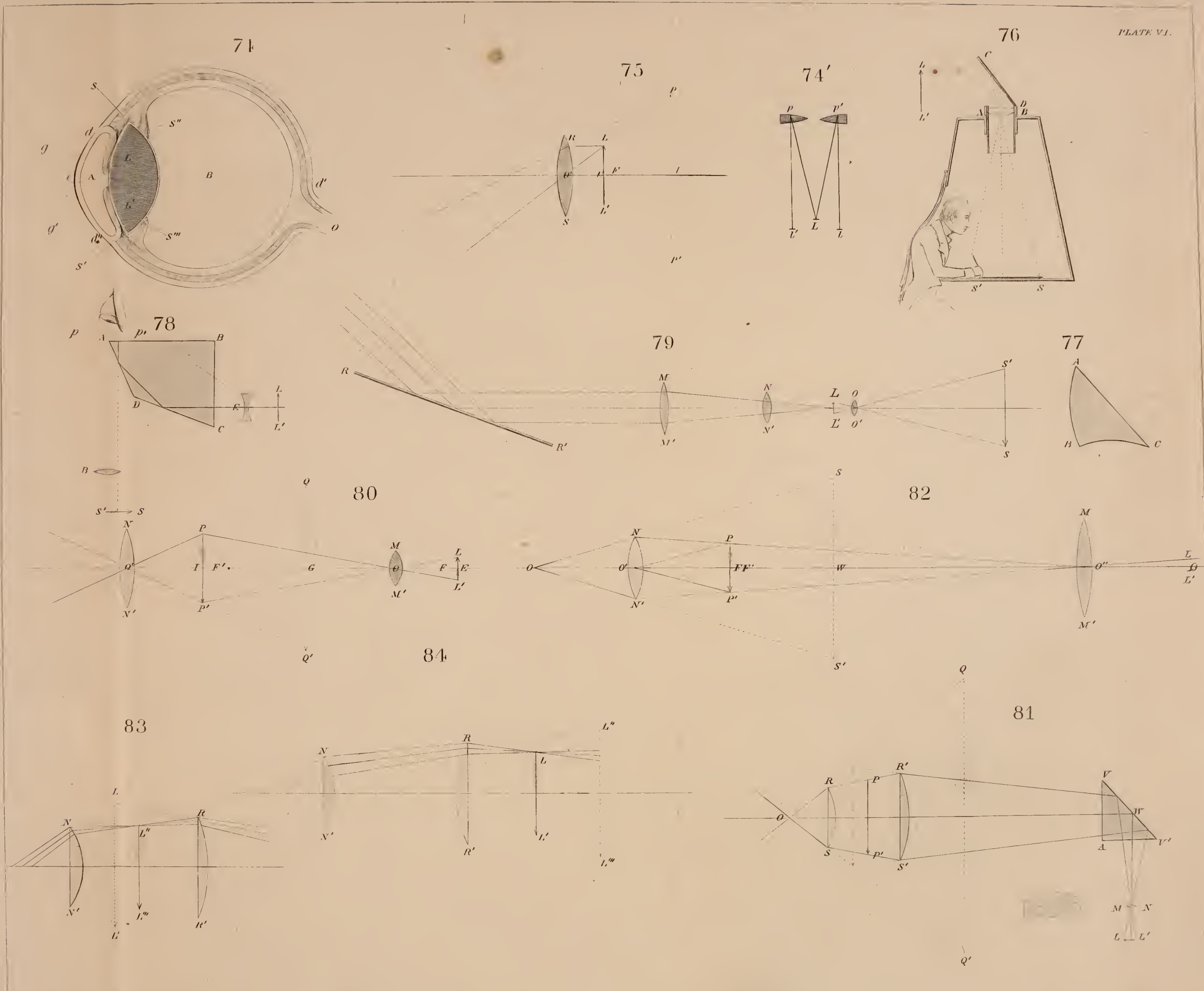


PLATE VI.



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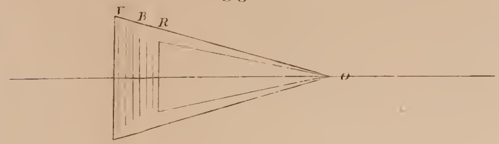

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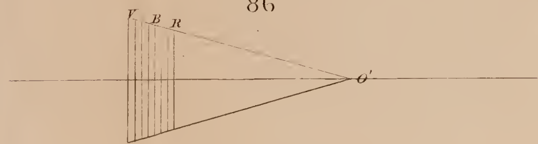

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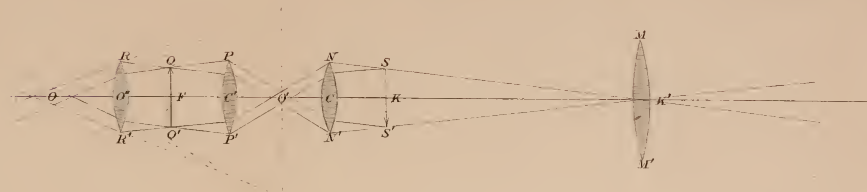

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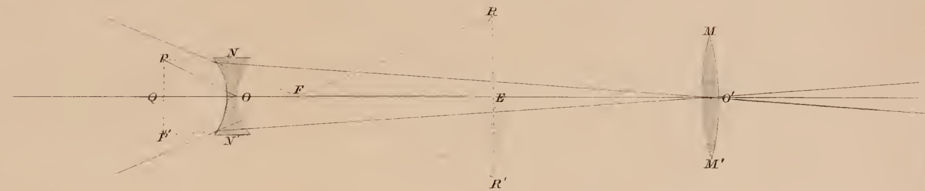

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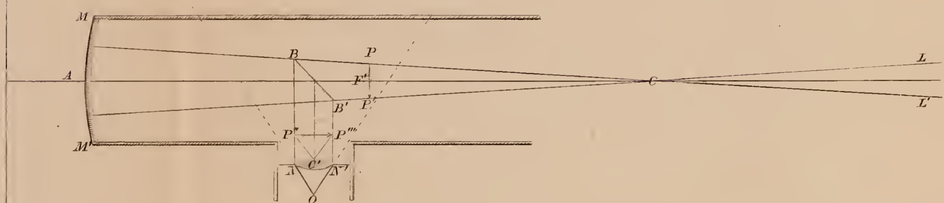

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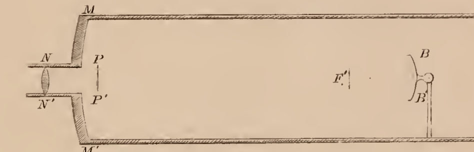

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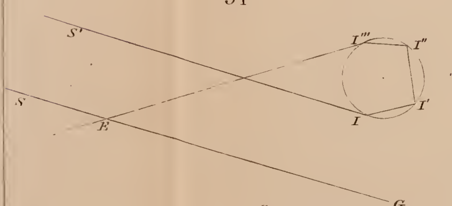

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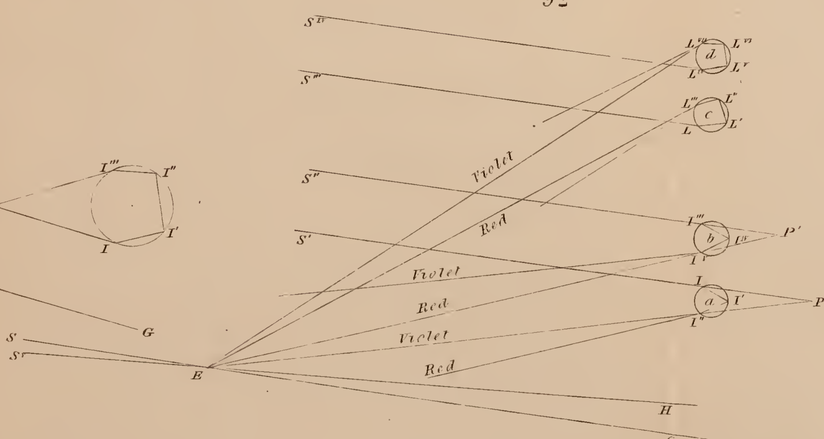

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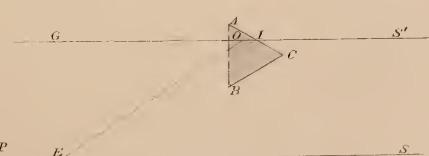

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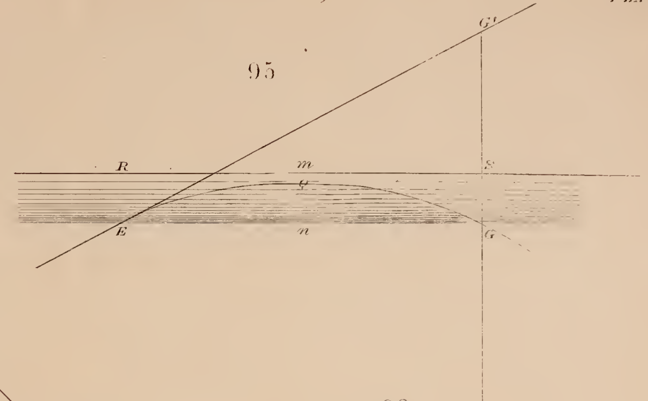

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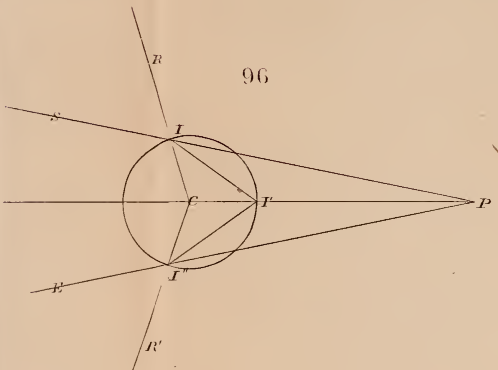

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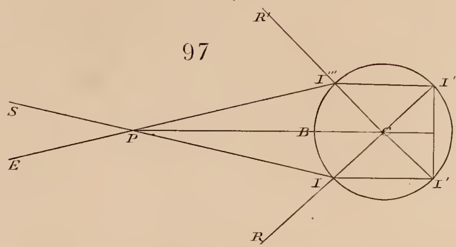

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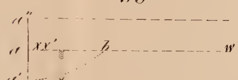

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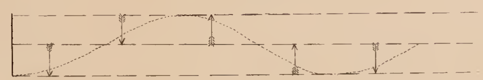

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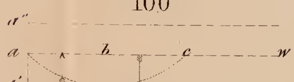

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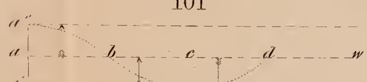

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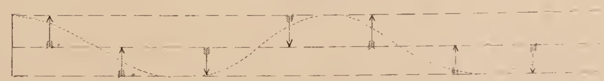

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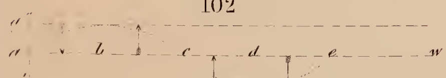

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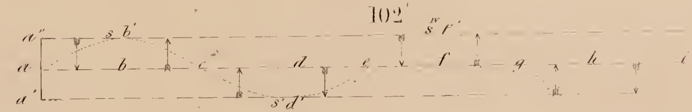

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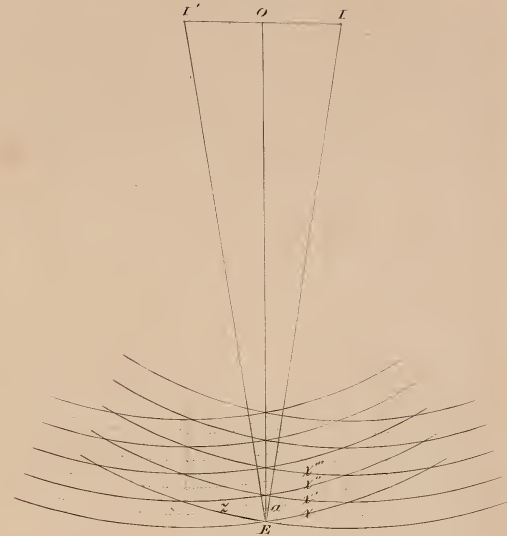

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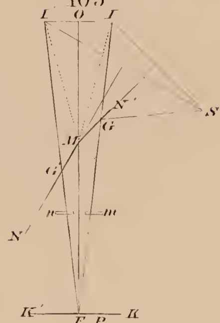

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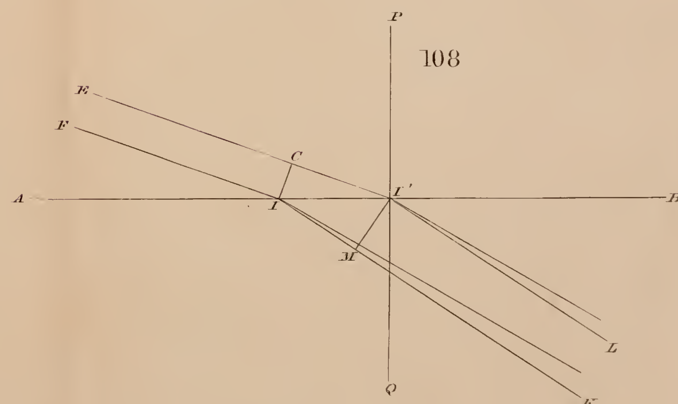

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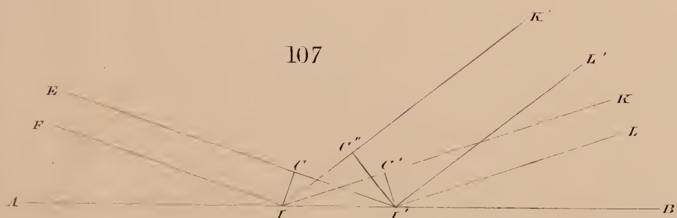

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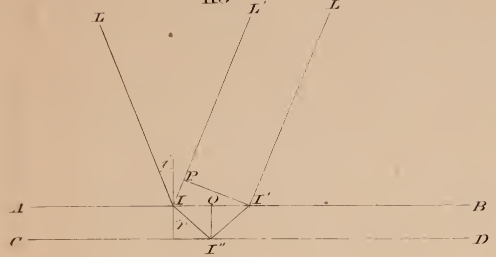

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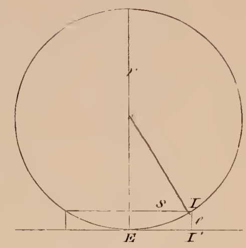

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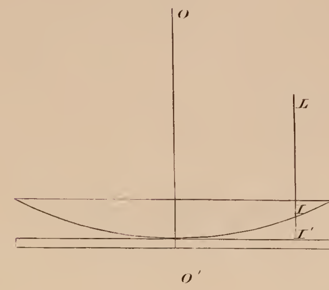

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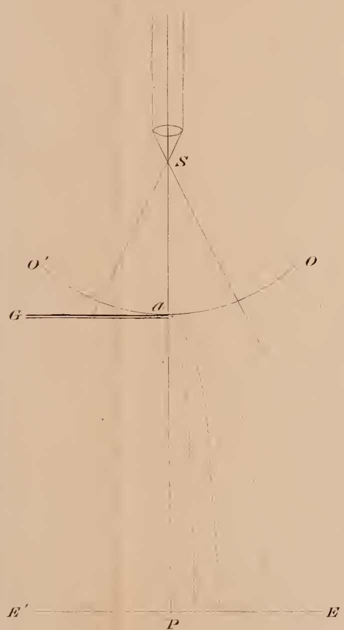

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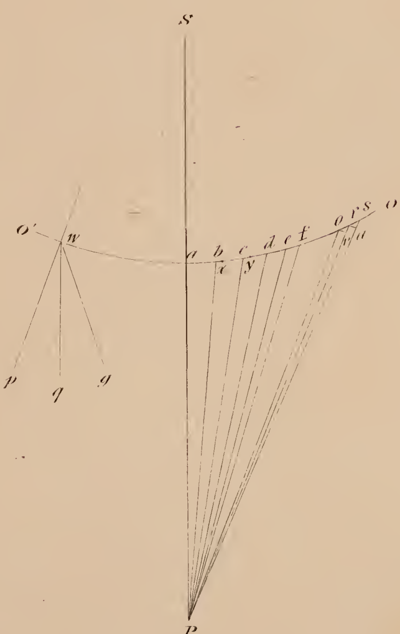

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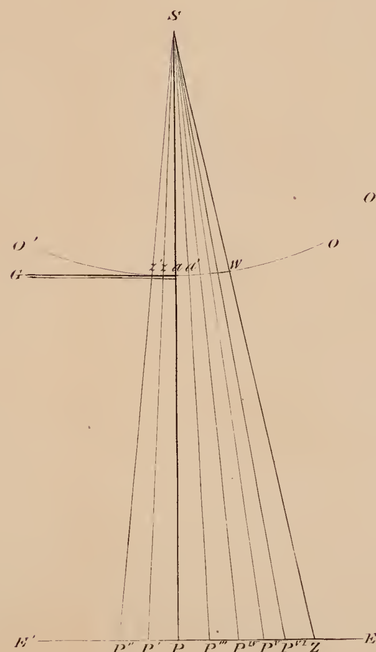

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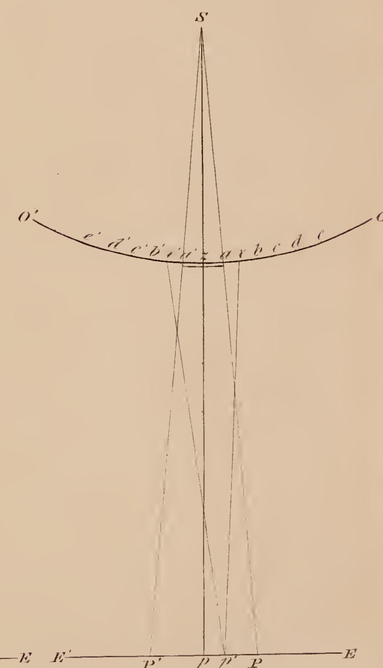
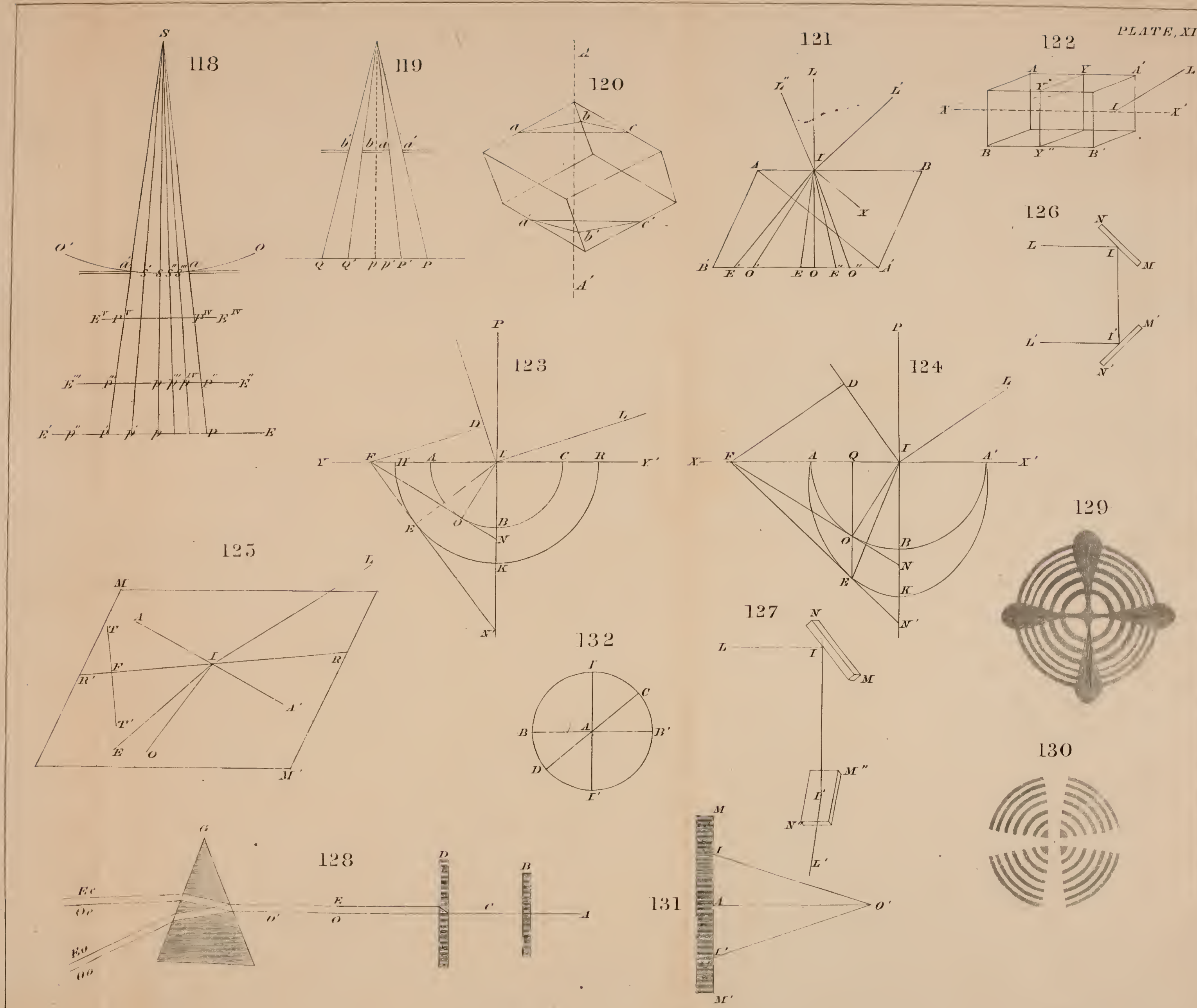
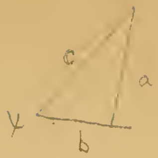


PLATE. XI.





Sin $\frac{a}{c}$

Cosin $\frac{b}{c}$

Sec $\frac{c}{b}$

Cosec $\frac{c}{a}$

Tan $\frac{a}{b}$

Cotan $\frac{b}{a}$

Trigonometrical Functions.

$\frac{a}{c} \times \frac{c}{b} = \frac{a}{b}$

Paul 792-816